

The Distance Constraint on Sequence Variables

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KU LEUVEN



Sequence Variables

Sequence Variables

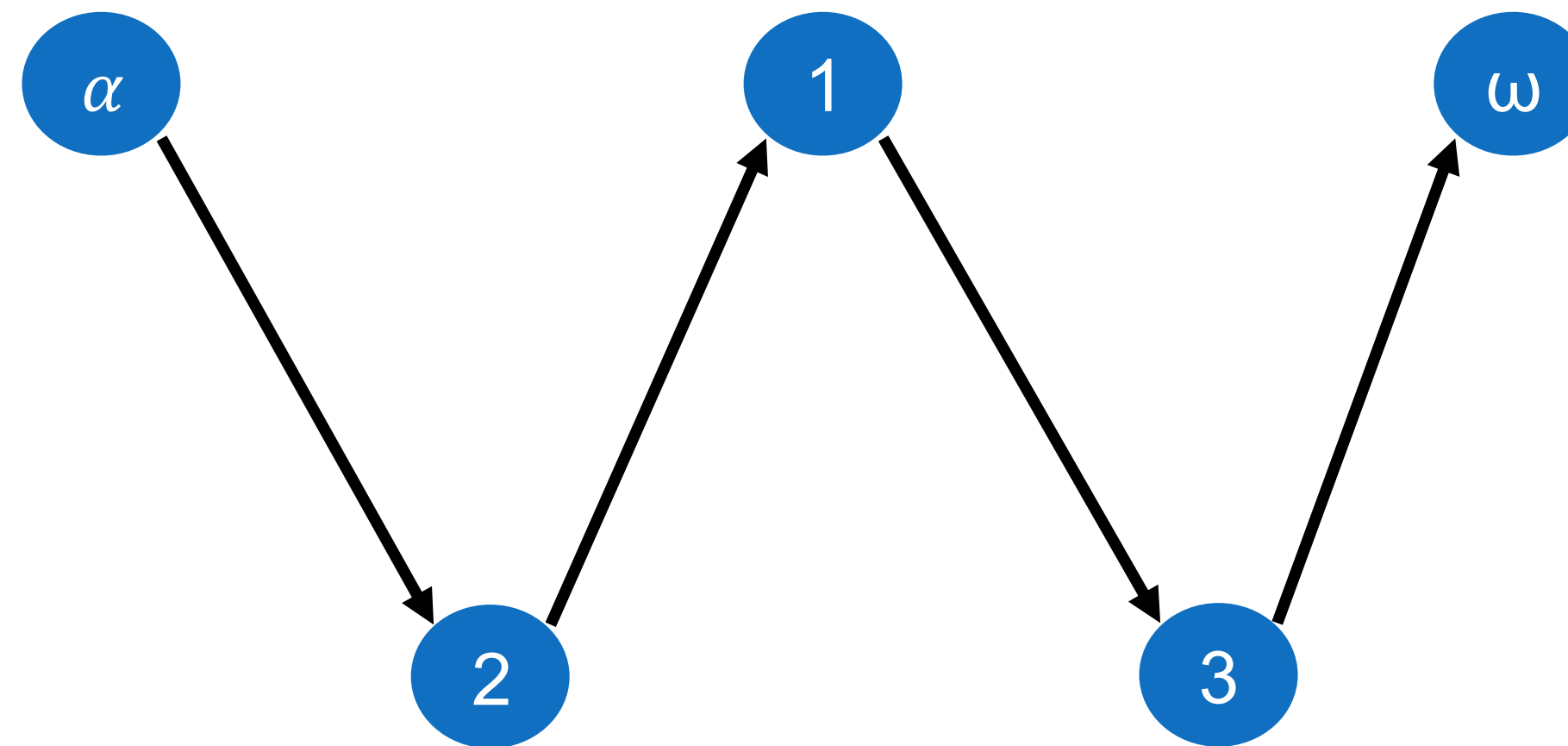
V a set of nodes



Sequence Variables

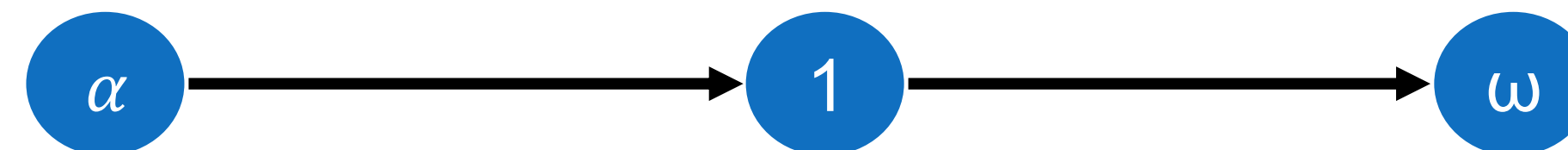
V a set of nodes

S a sequence



Sequence Variables

V a set of nodes

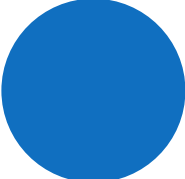


S a sequence

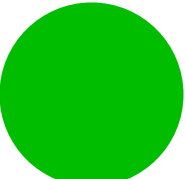
s a subsequence

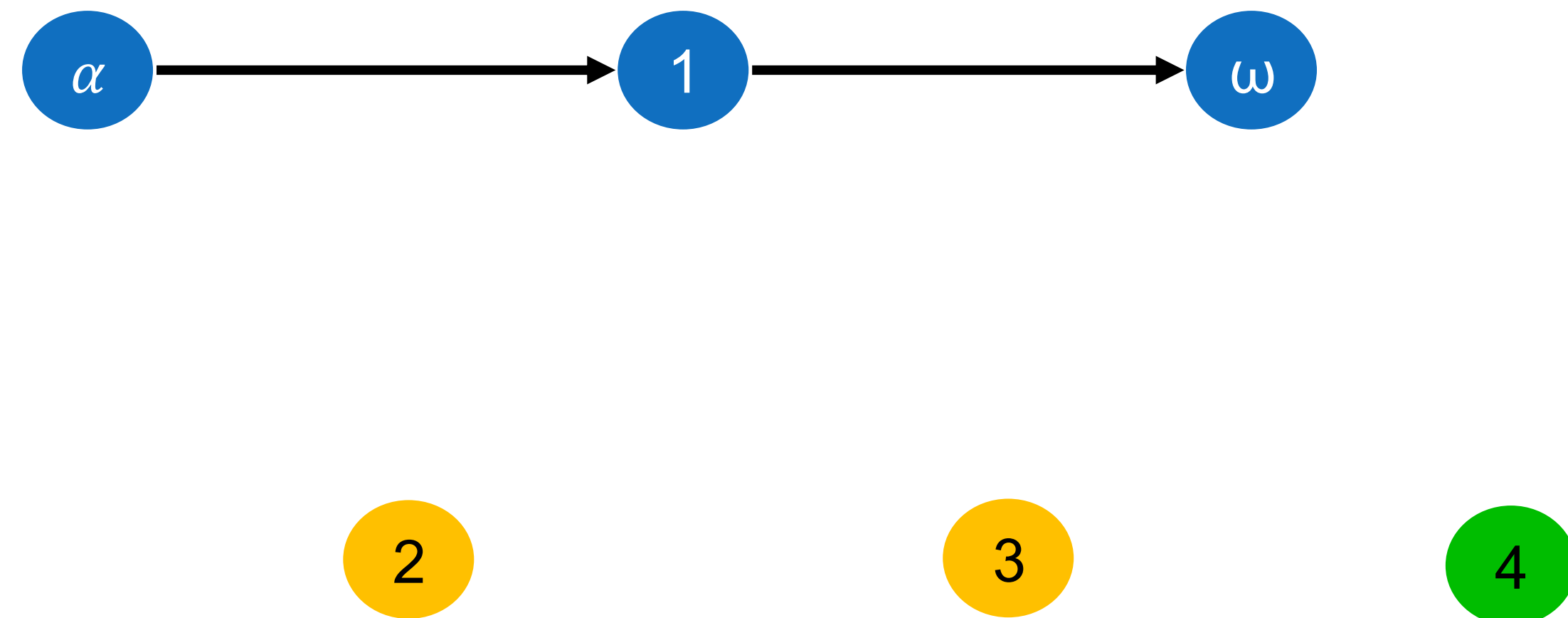


Sequence Variables

 Subsequence node

 Required node

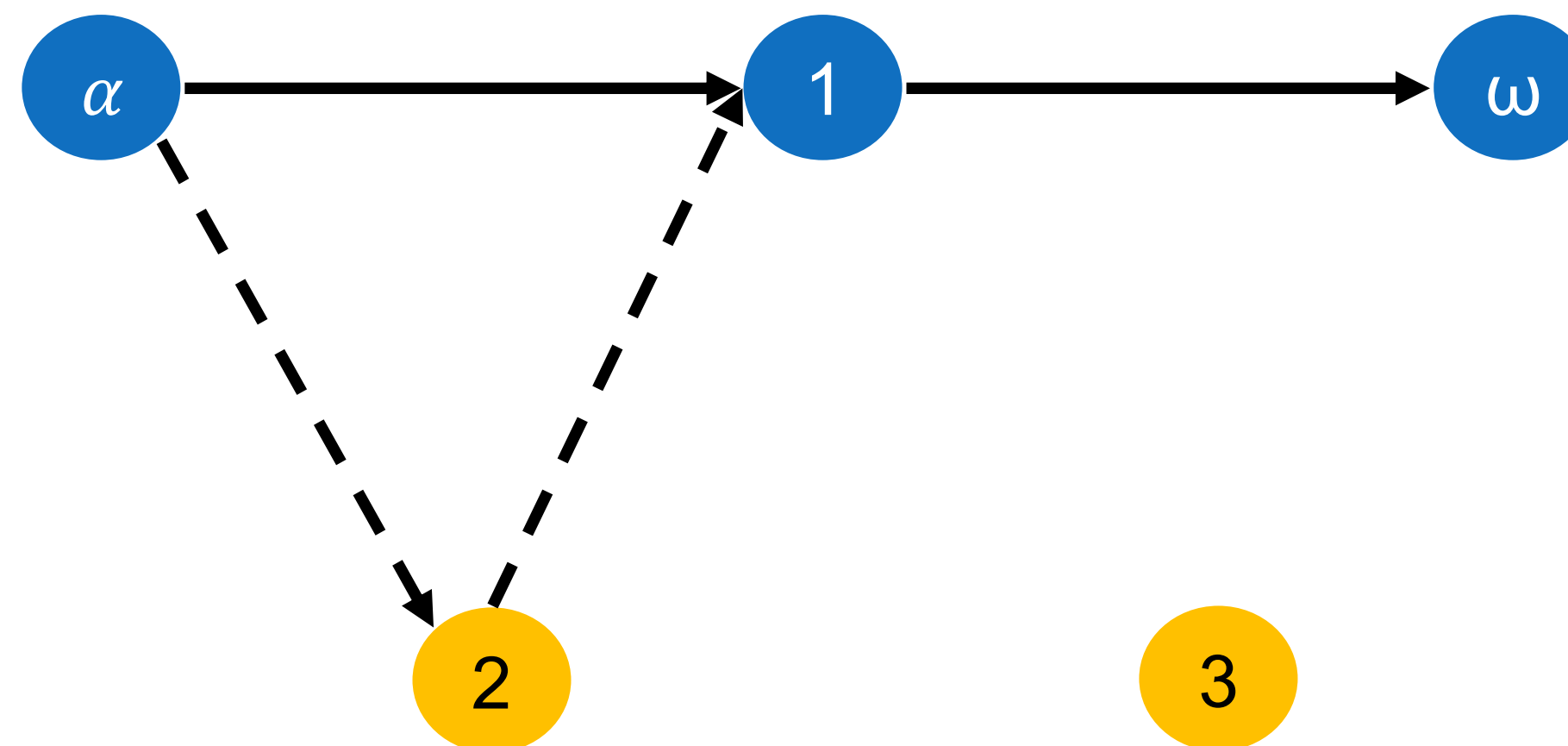
 Optional node



Sequence Variables

● Subsequence node

● Required node

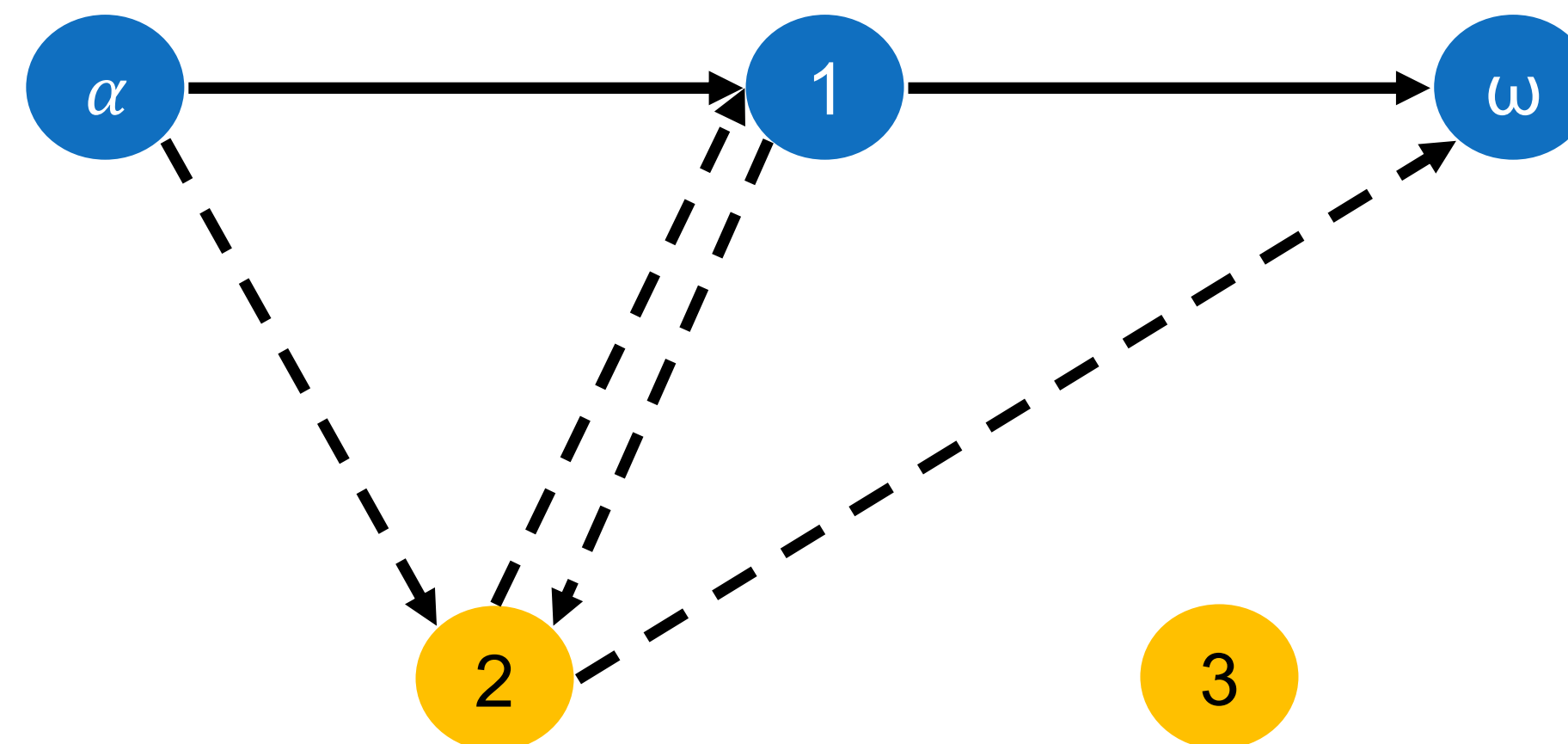


The Distance Constraint on Sequence Variables

Sequence Variables

● Subsequence node

● Required node

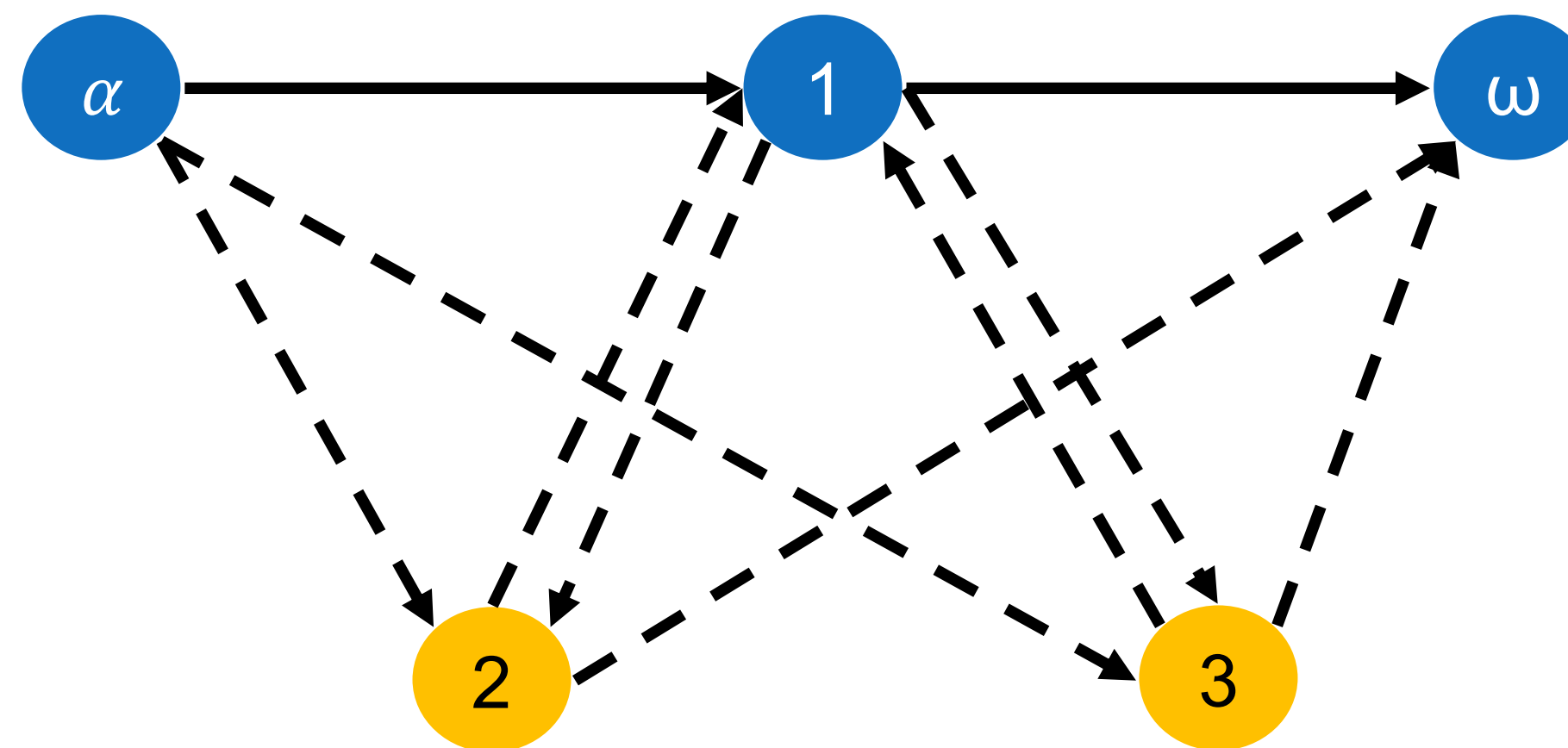


The Distance Constraint on Sequence Variables

Sequence Variables

● Subsequence node

● Required node

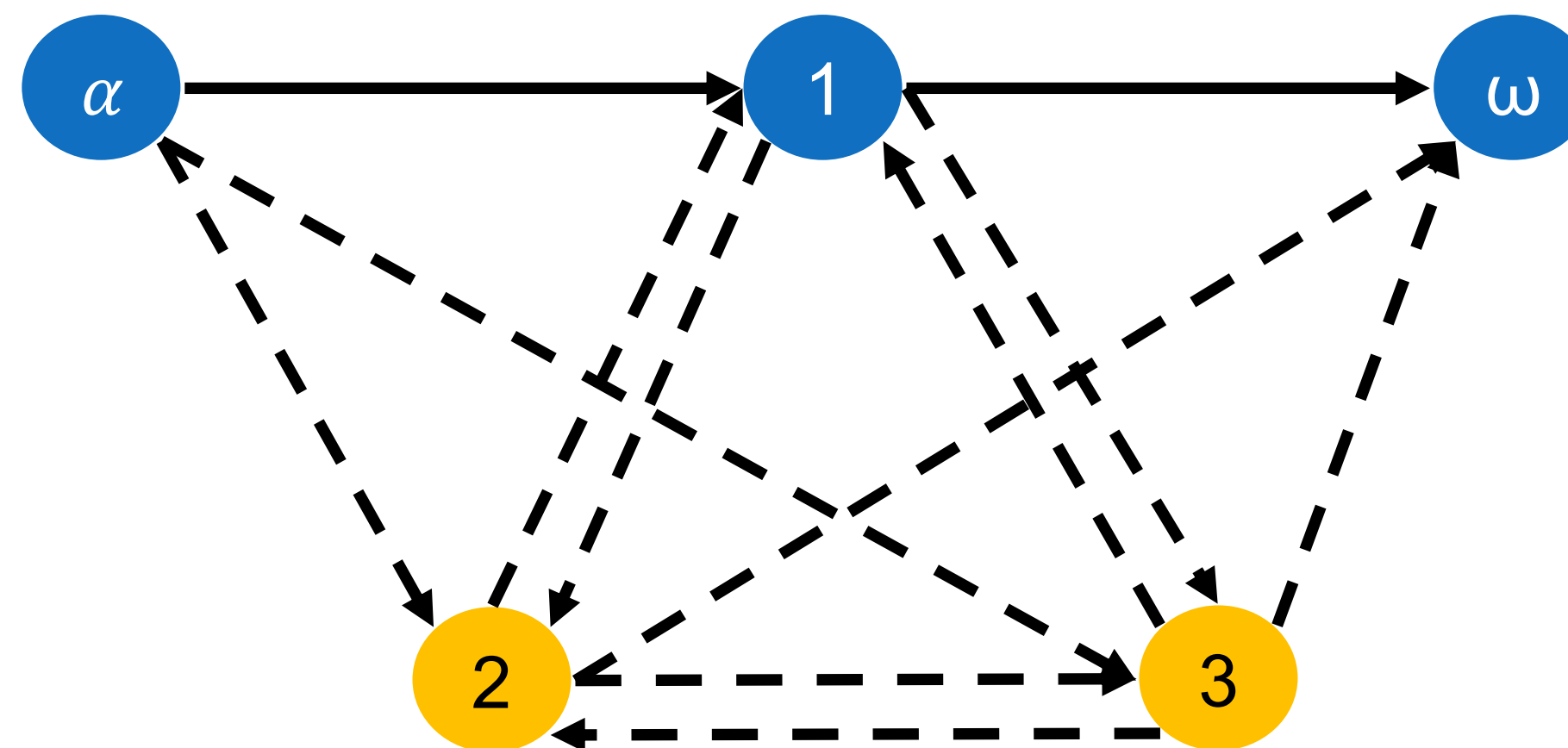


The Distance Constraint on Sequence Variables

Sequence Variables

● Subsequence node

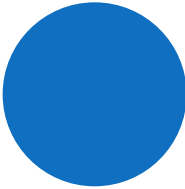
● Required node



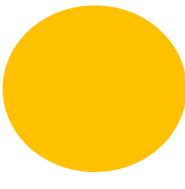
The Distance Constraint on Sequence Variables

Distance Constraint

Distance Constraint

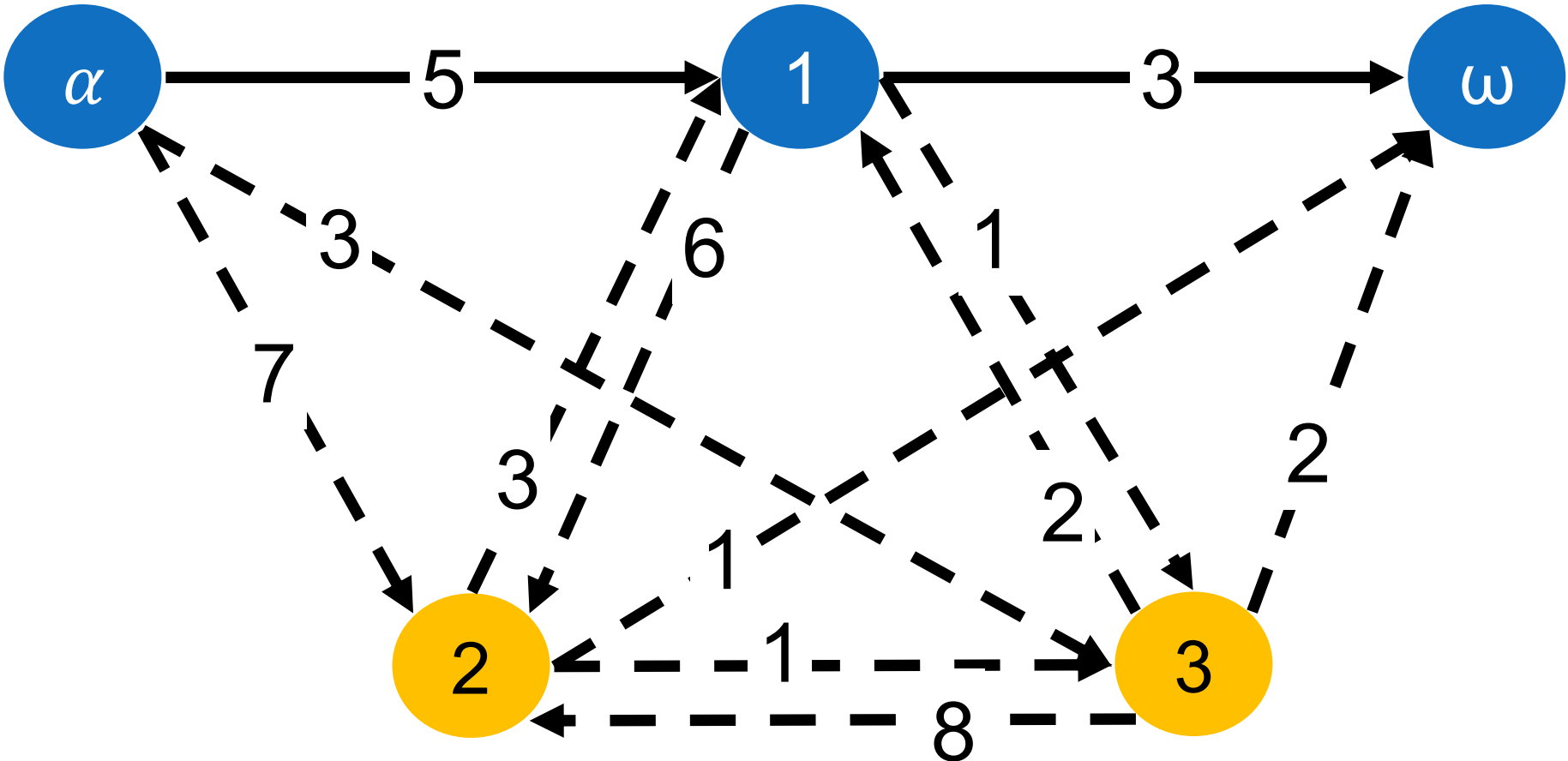


Subsequence node



Required node

$\text{Cost}(S) \leq \text{Dist} = 12$



Triangular inequality

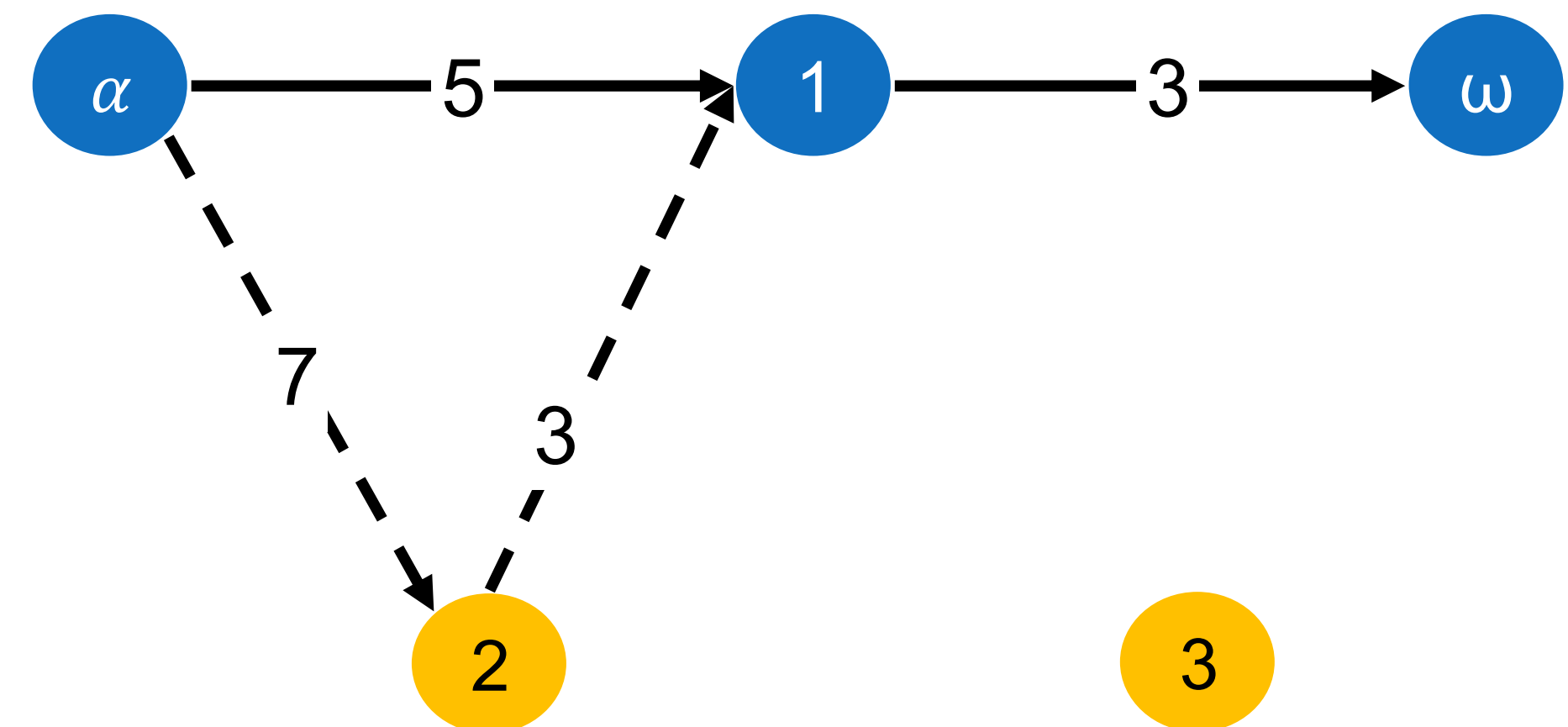
Distance Constraint

● Subsequence node

● Required node

$$\Delta(\alpha, 2, 1) = \text{cost}(\alpha, 2) + \text{cost}(2, 1) - \text{cost}(\alpha, 1)$$

$$\Delta(\alpha, 2, 1) = 7 + 3 - 5 = 5$$



Actual Filtering

Actual Filtering

Remove all the insertion that cannot be part of a solution is NP-complet



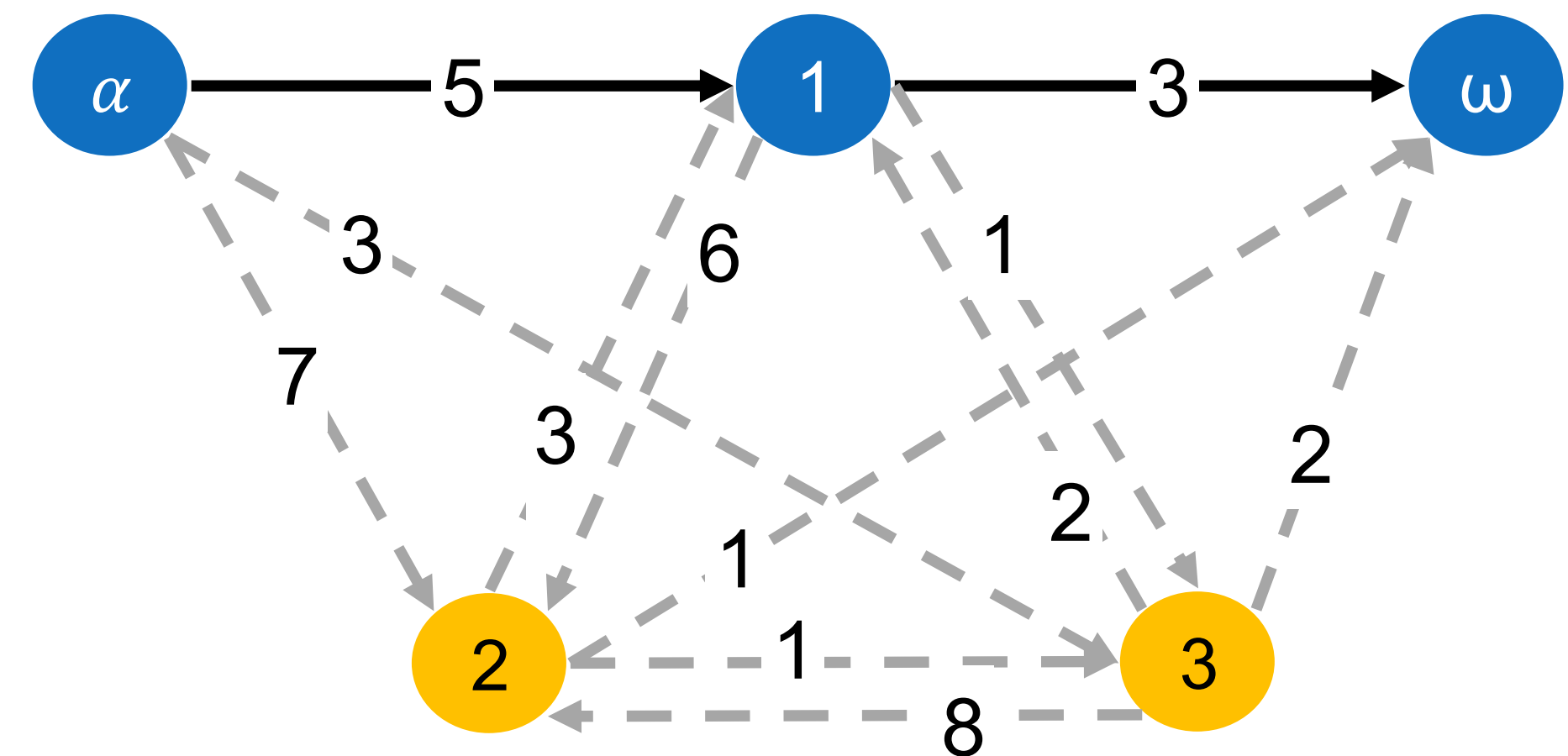
We use lower bound



We do not remove ALL the insertion that cannot be part of a solution

Actual Filtering

$$L_O = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$
$$L_O \leq \text{Dist}$$
$$8 \leq 12$$

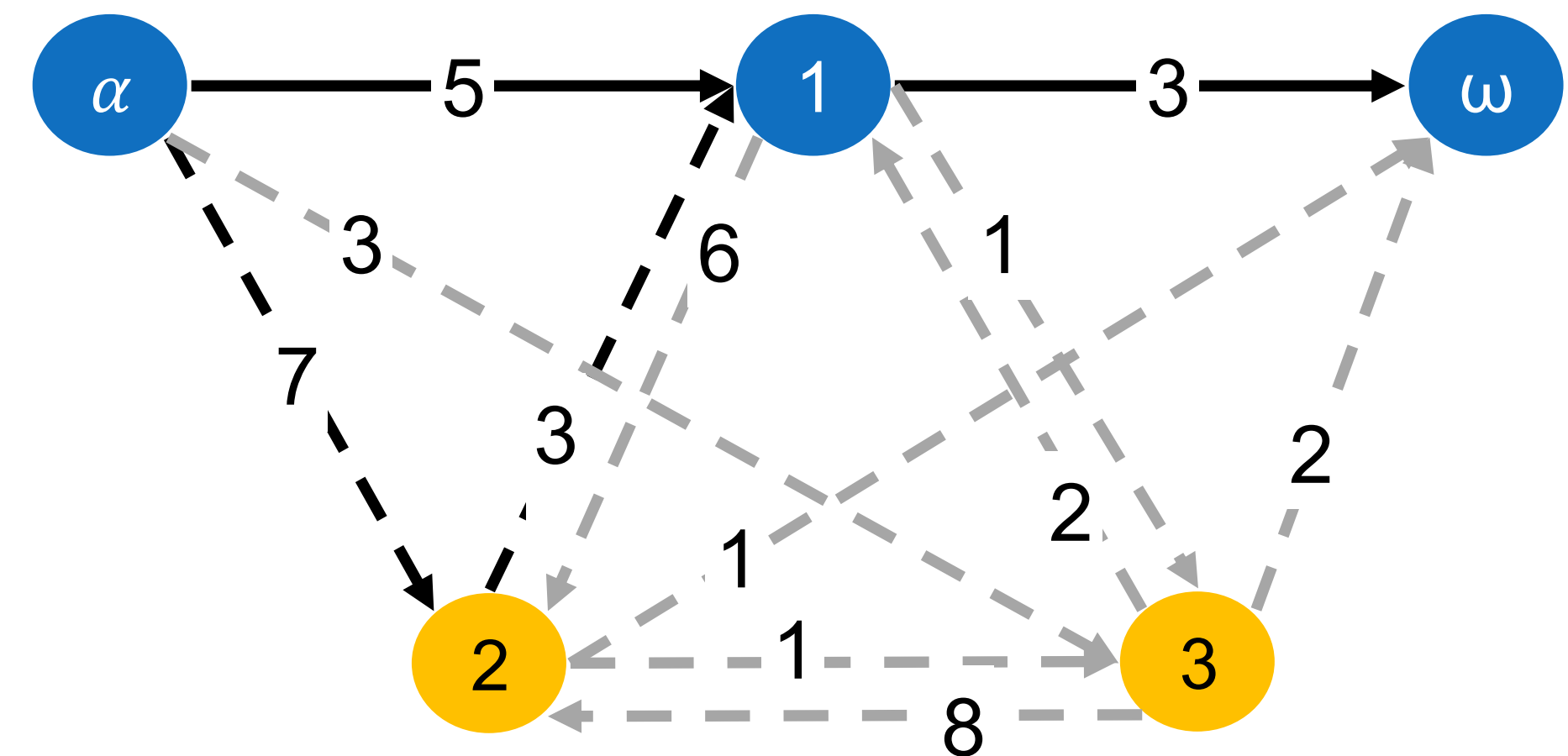


Actual Filtering

$$L_O = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$
$$L_O \leq \text{Dist}$$
$$8 \leq 12$$

Filtrage

$$L_O + \Delta(\alpha, 2, 1) = 13 > \text{Dist}$$

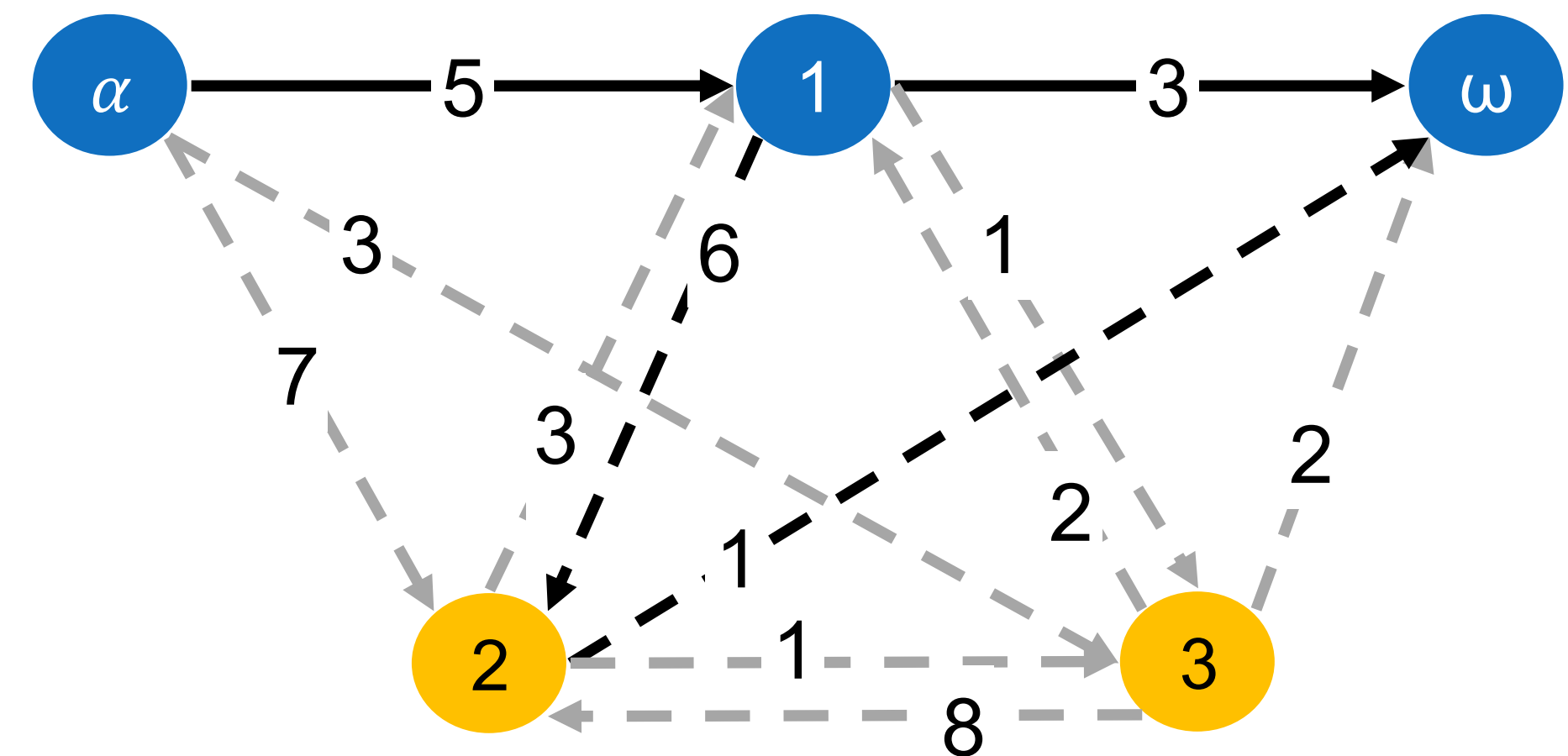


Actual Filtering

$$L_O = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$
$$L_O \leq \text{Dist}$$
$$8 \leq 12$$

Filtrage

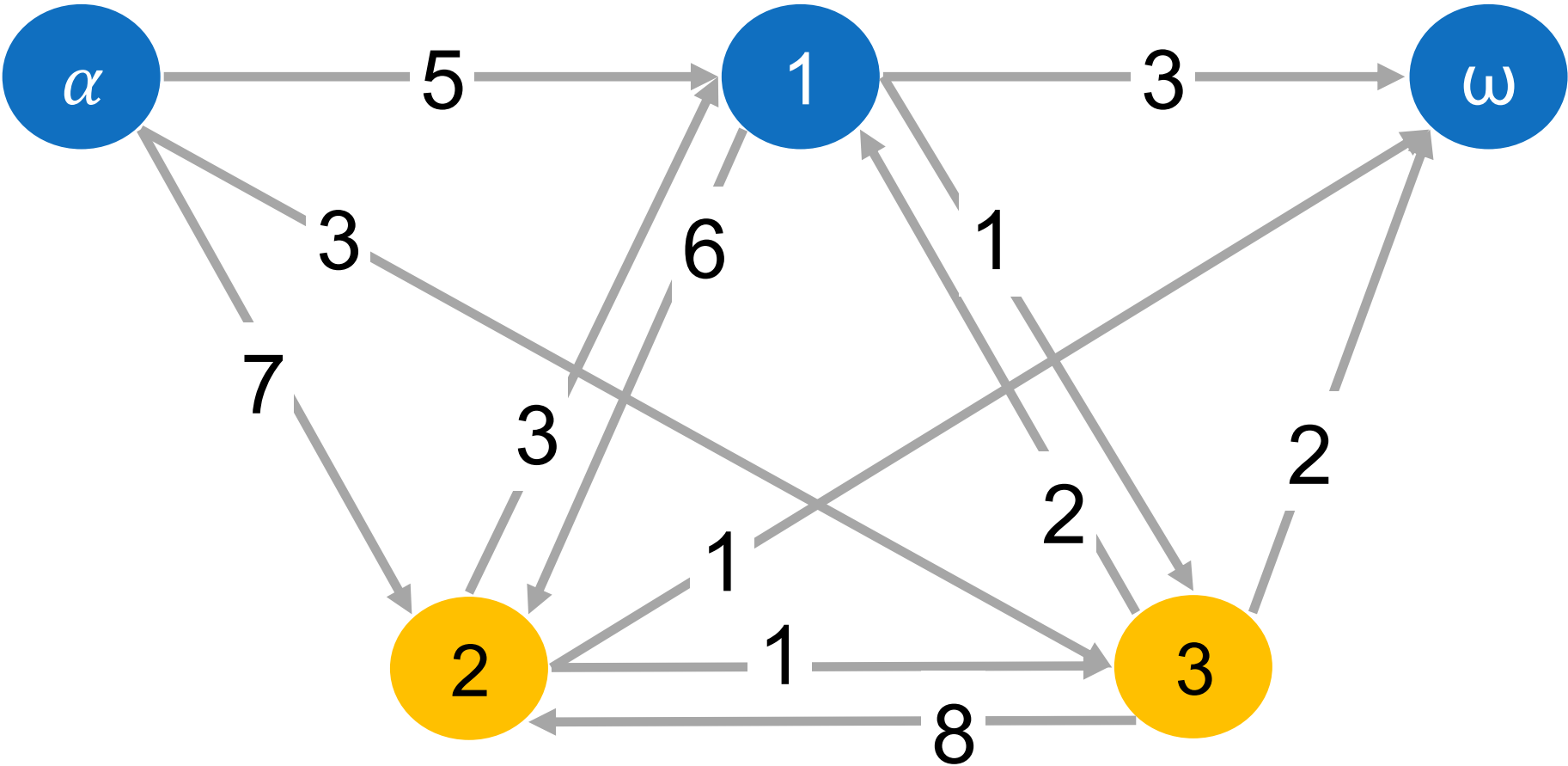
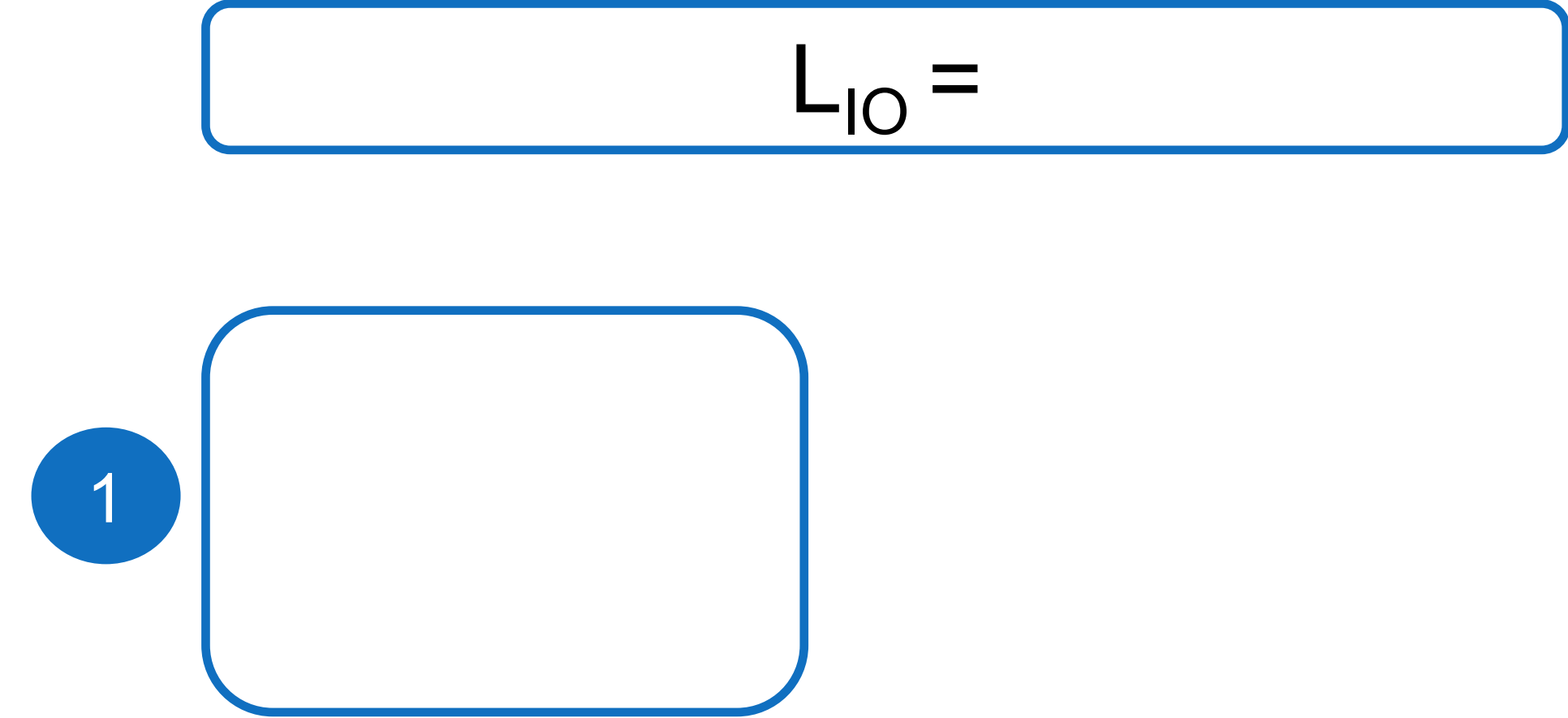
$$L_O + \Delta(1, 2, \omega) = 12 \leq \text{Dist}$$



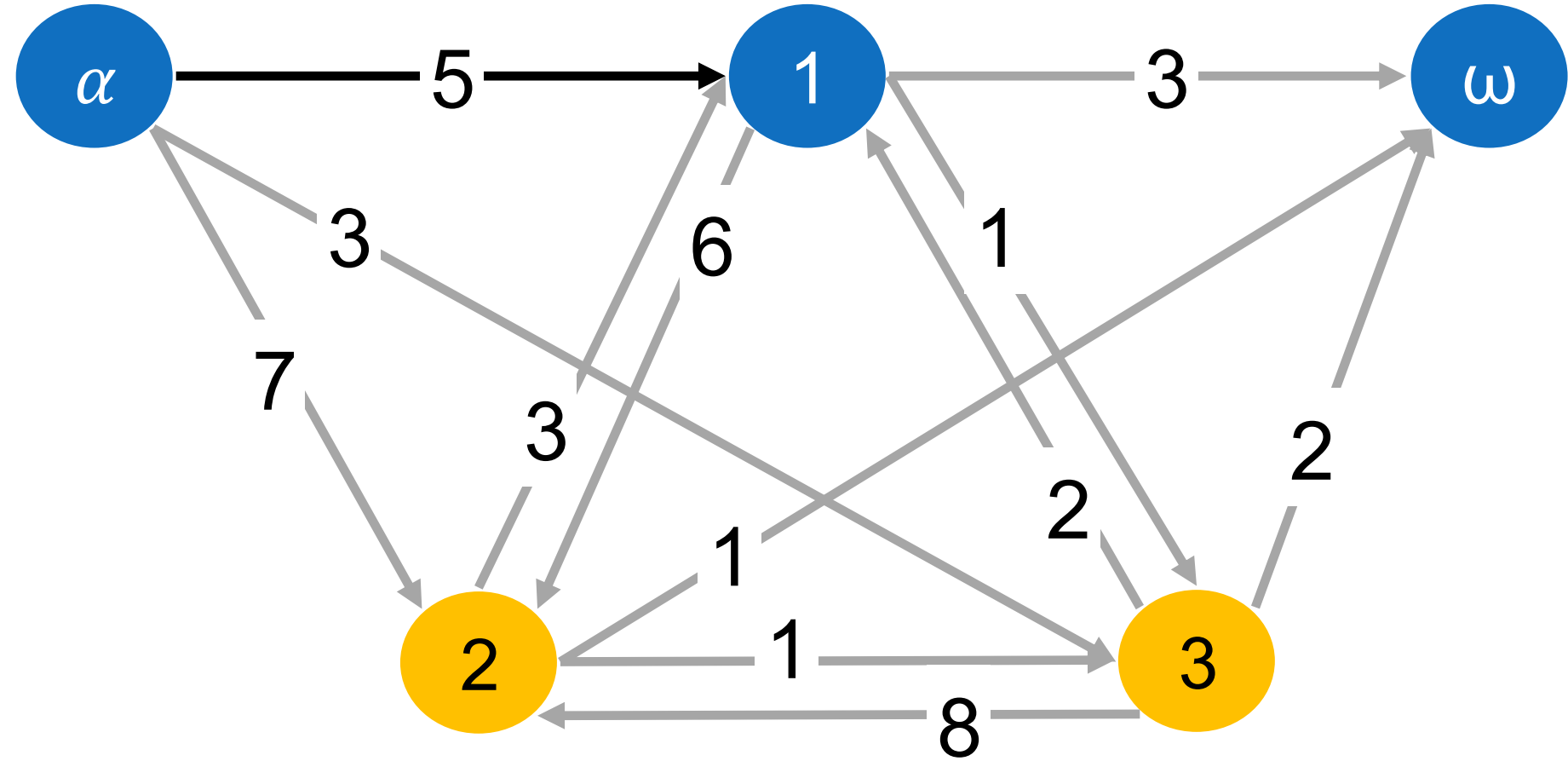
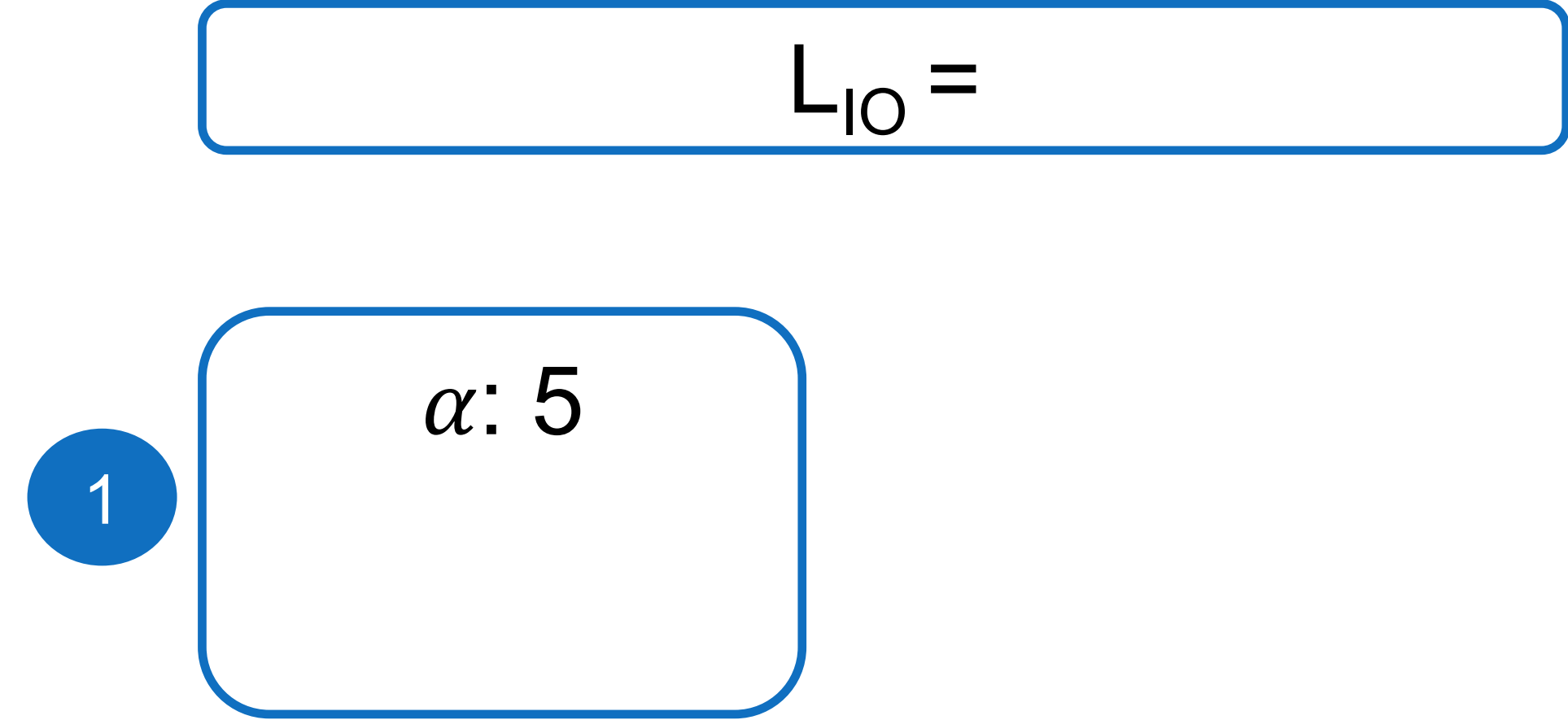
Lower bound based on the inputs

And outputs

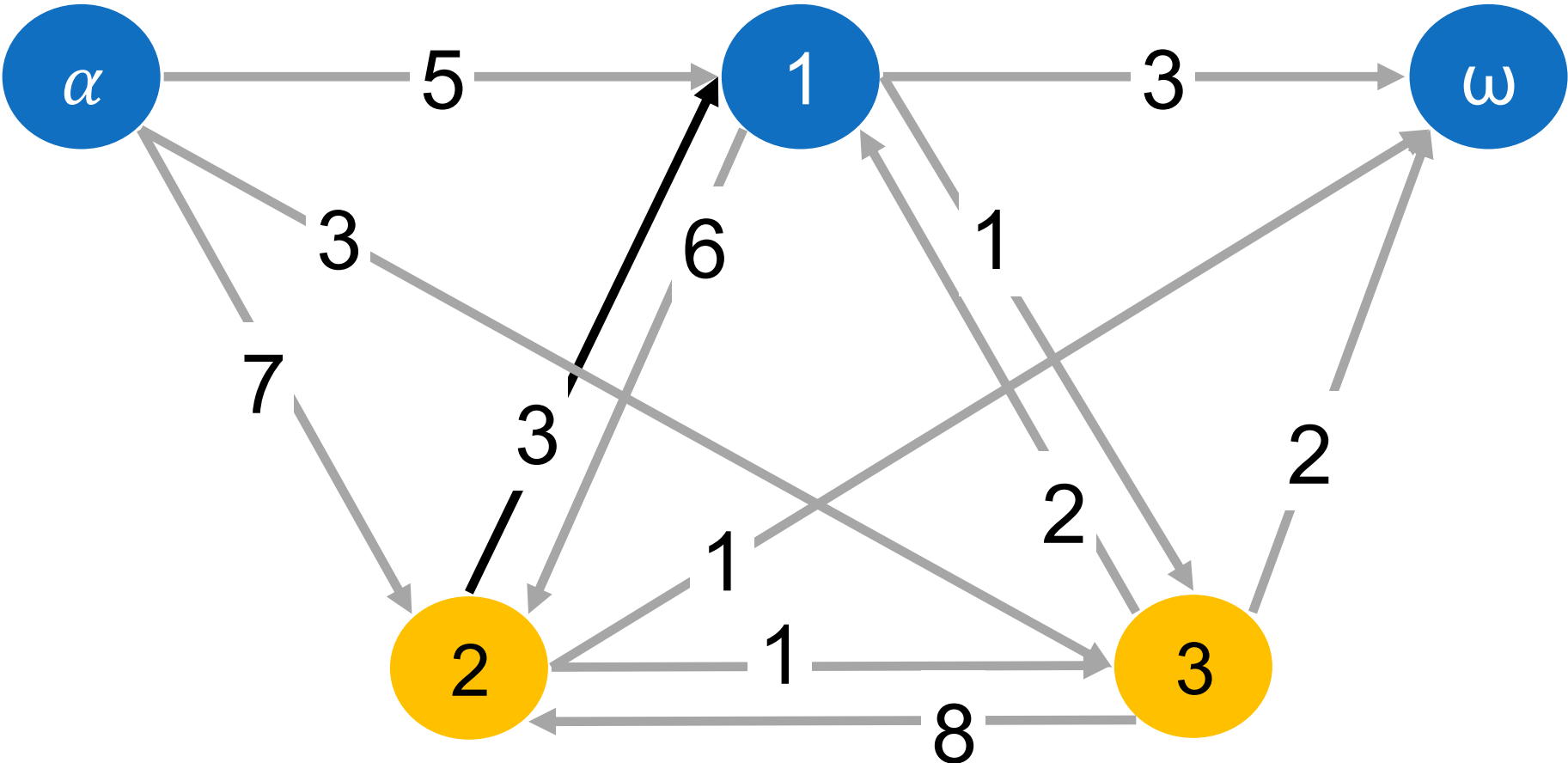
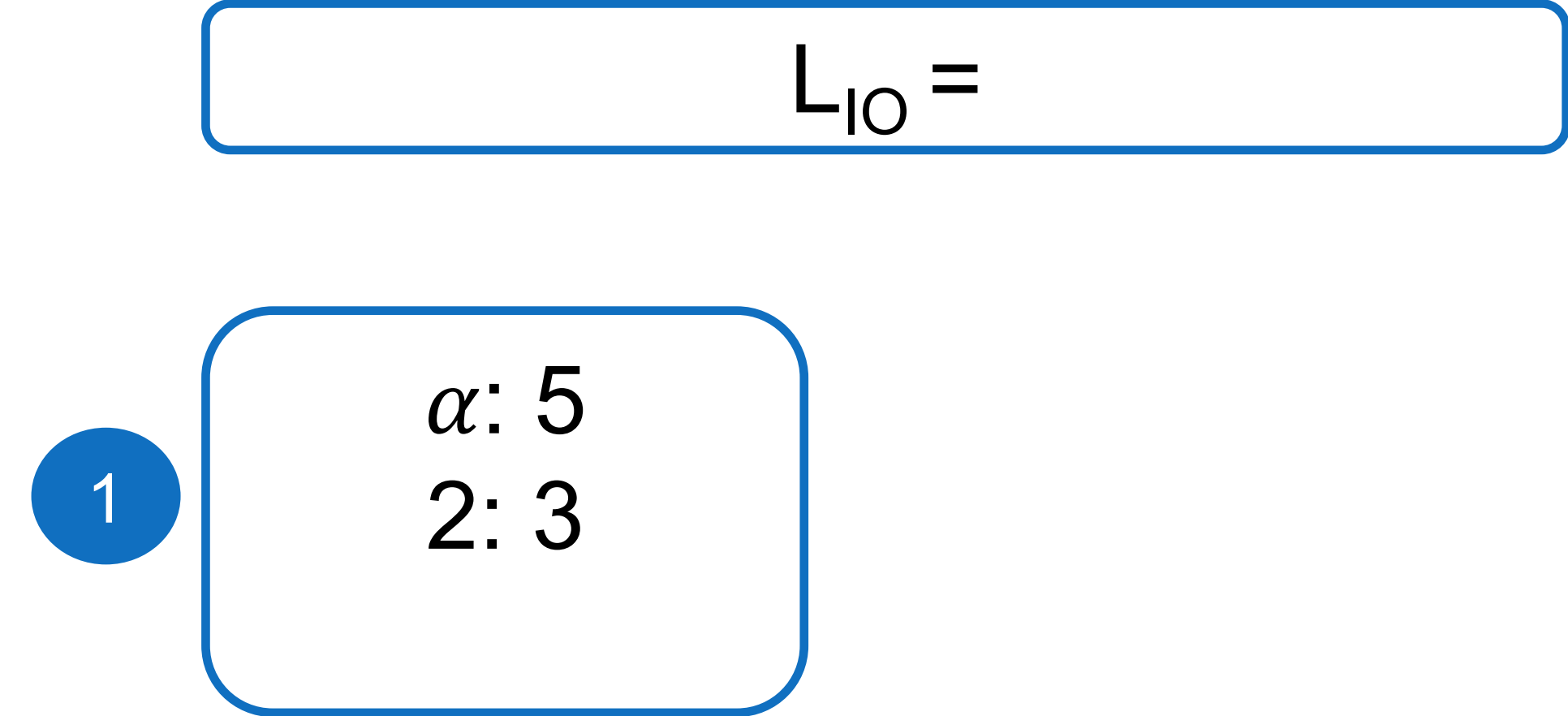
Lower bound based on the inputs



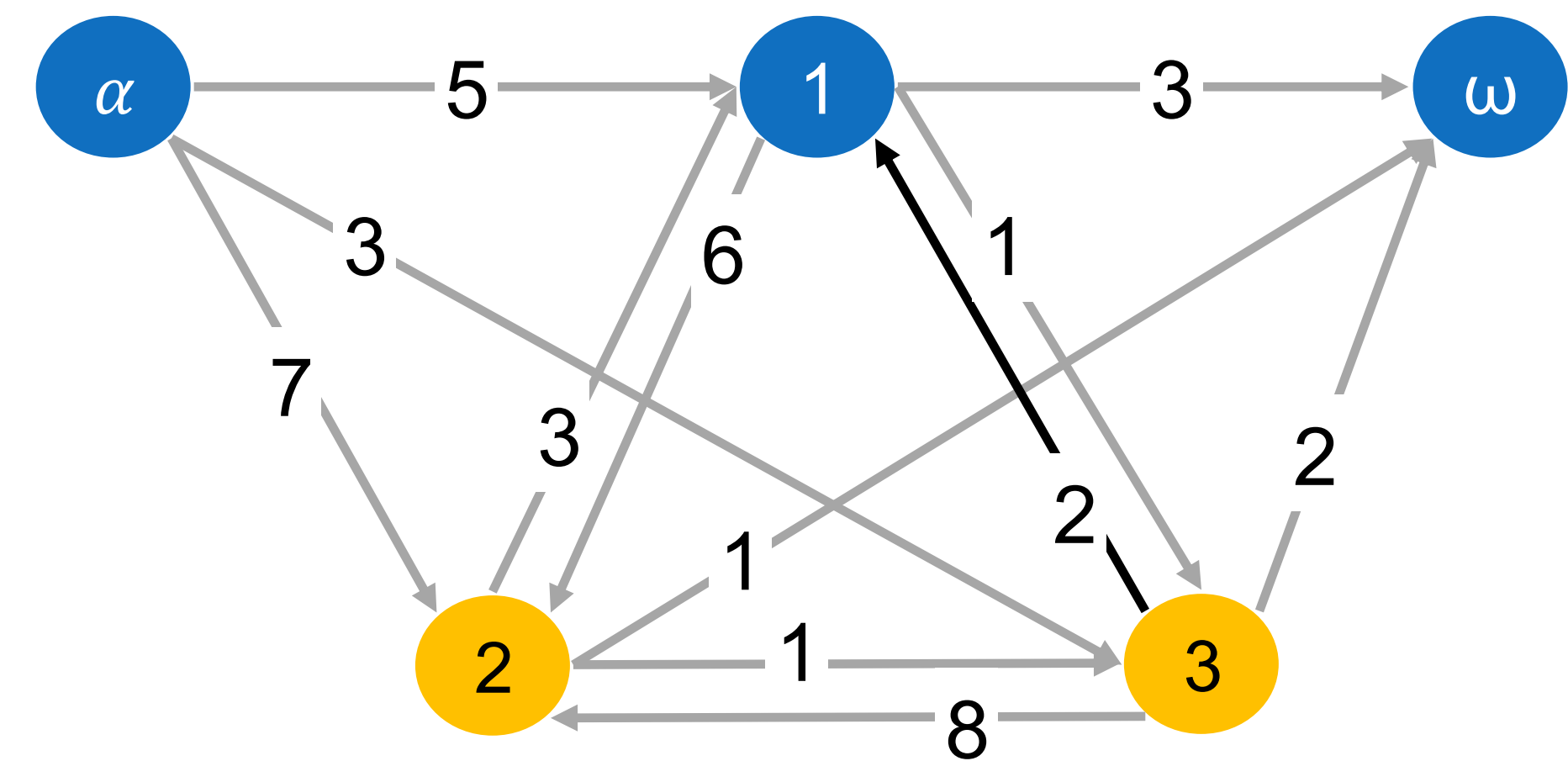
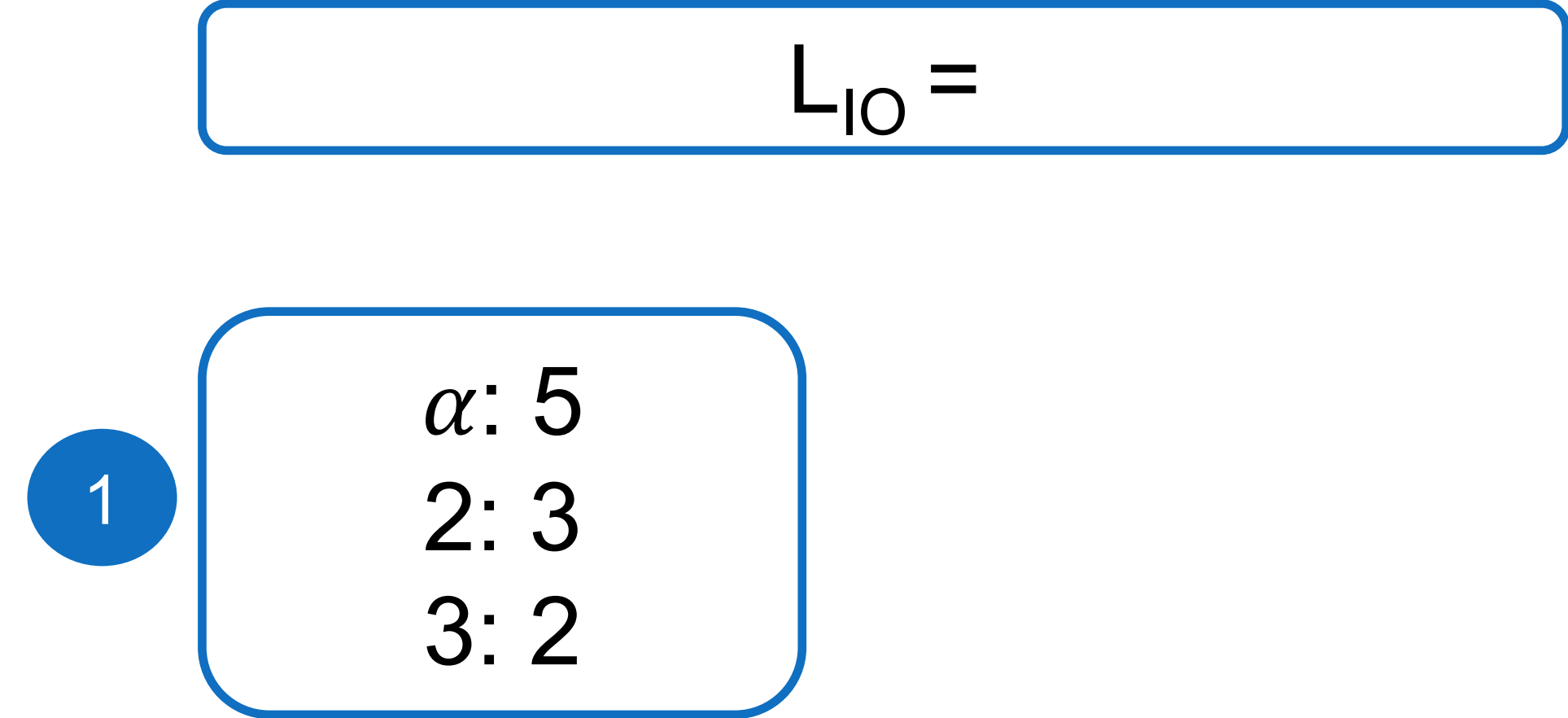
Lower bound based on the inputs



Lower bound based on the inputs

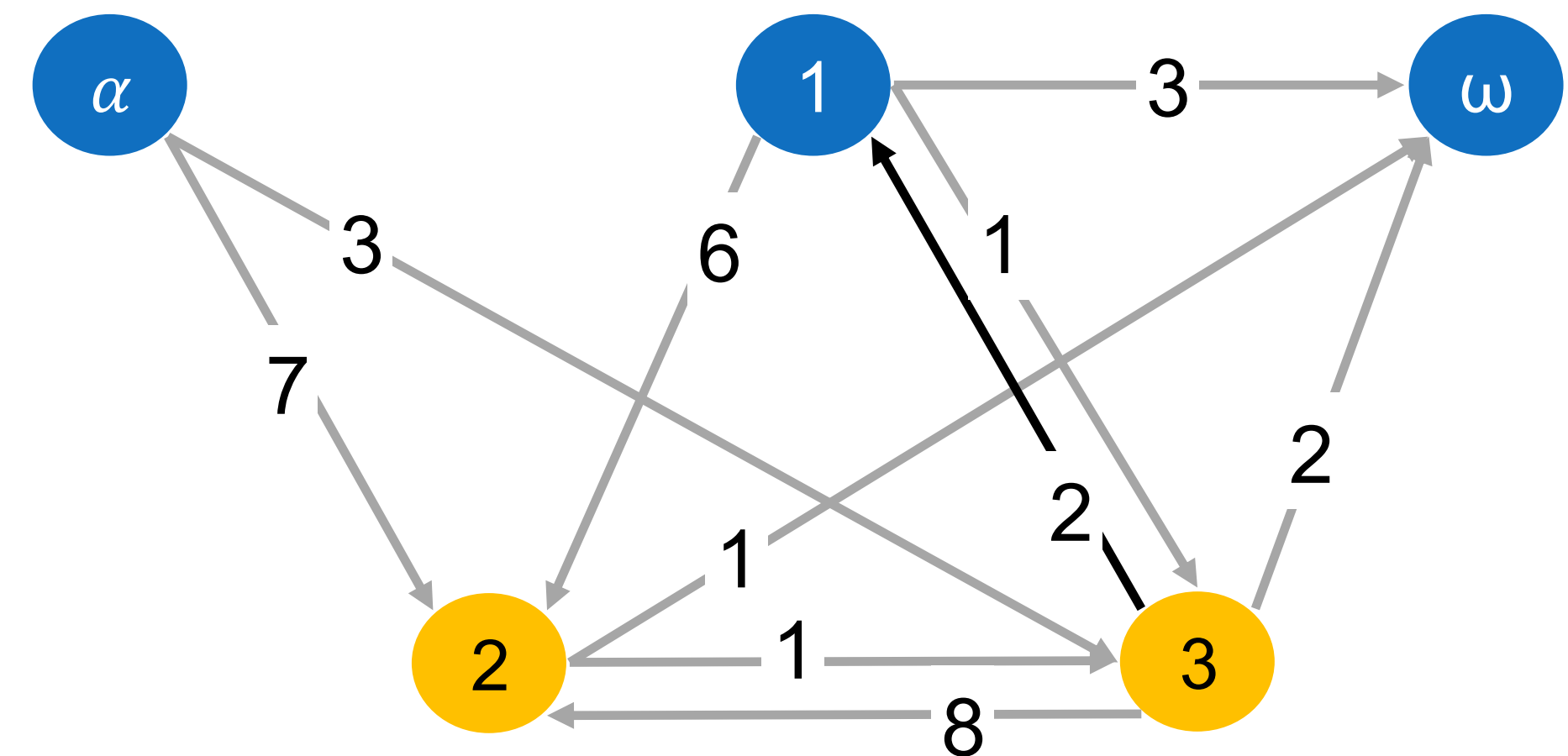
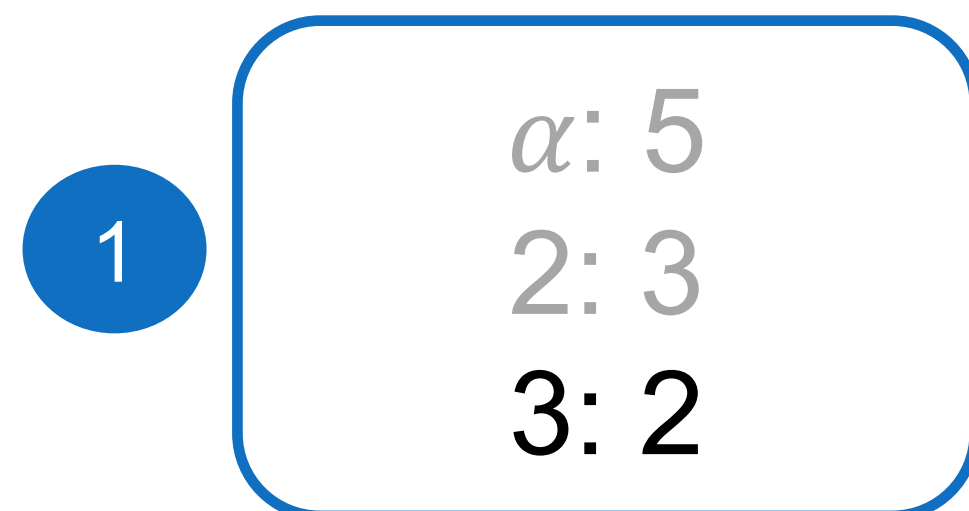


Lower bound based on the inputs



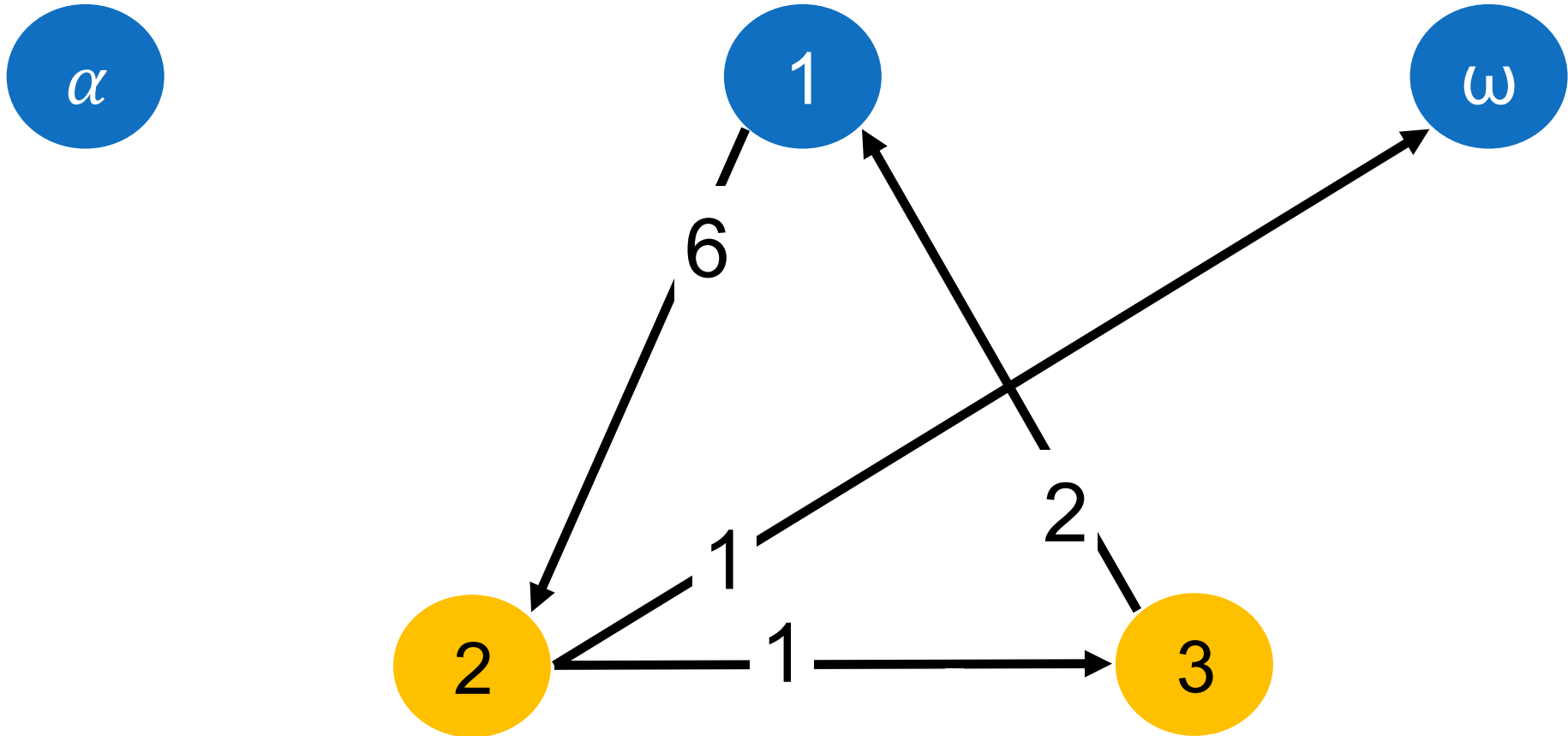
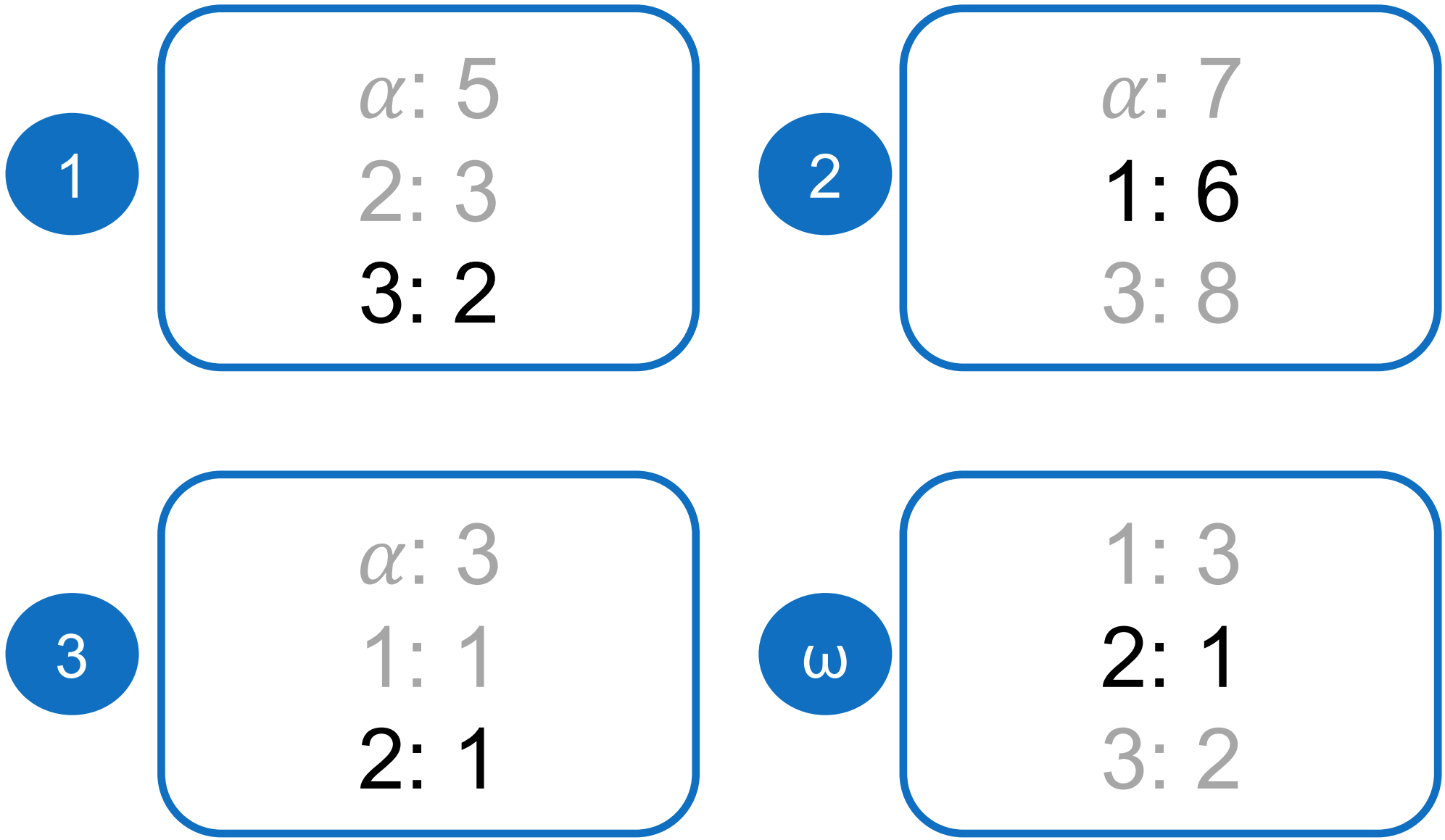
Lower bound based on the inputs

$$L_{IO} = 2 +$$



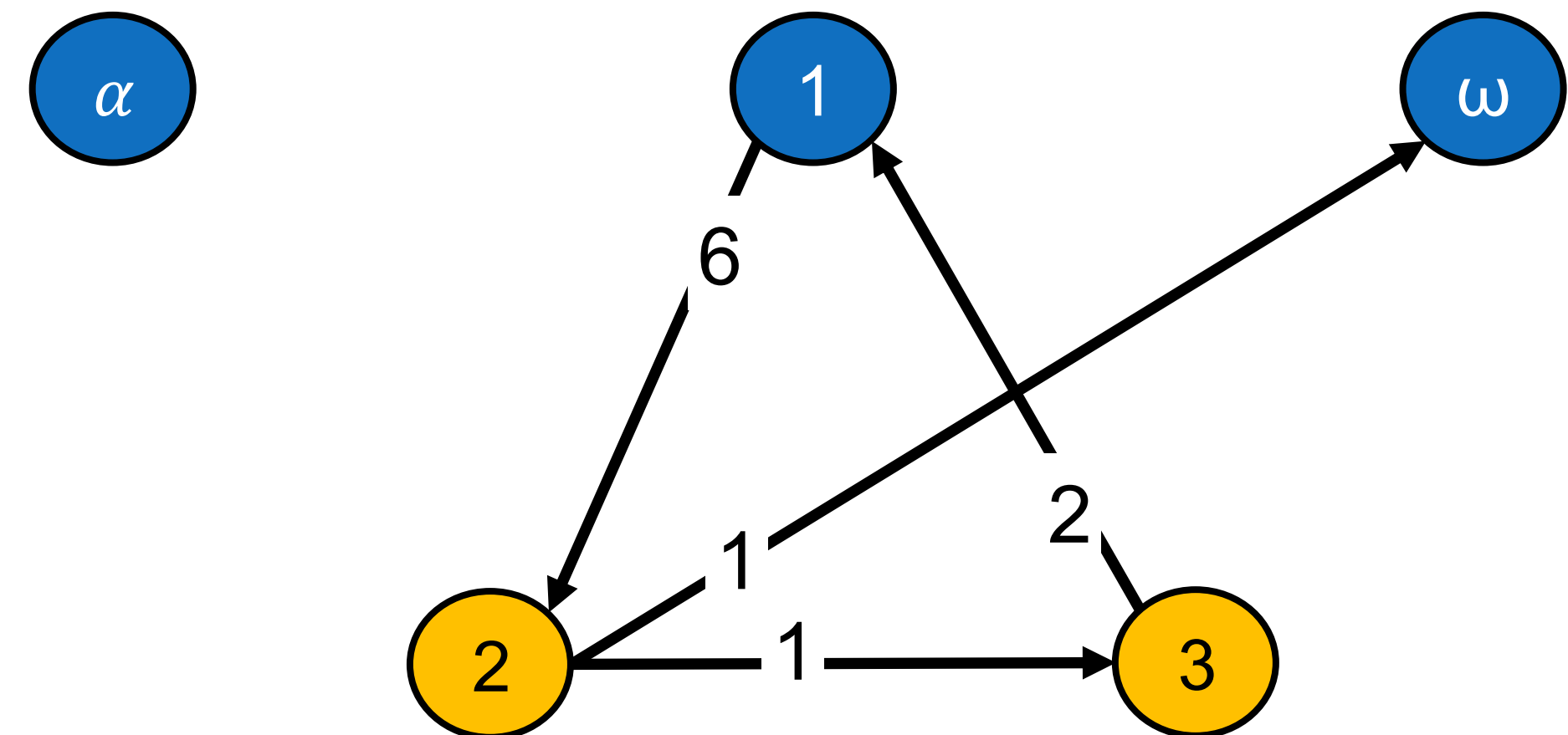
Lower bound based on the inputs

$$L_{IO} = 2 + 6 + 1 + 1 = 10$$



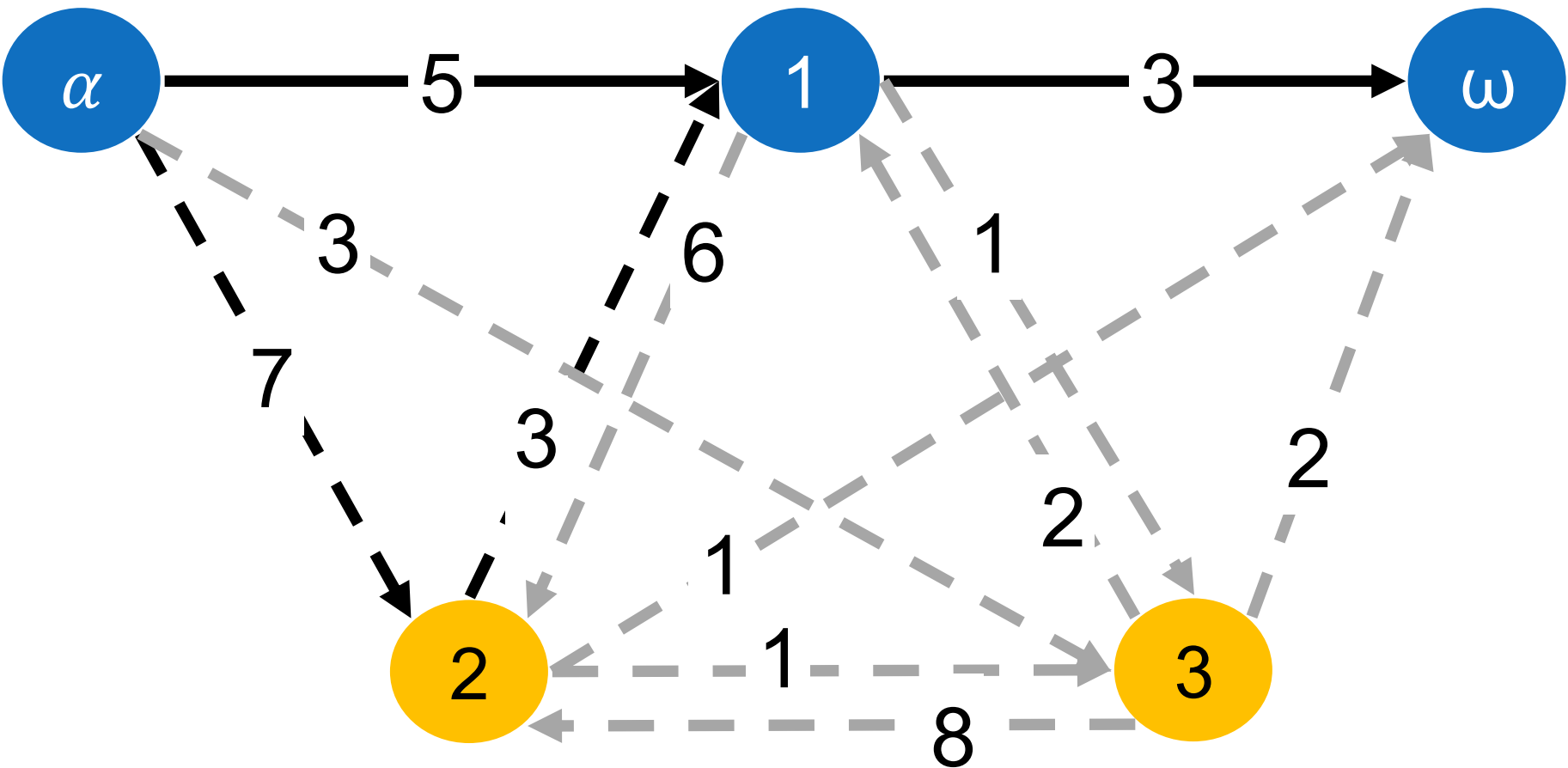
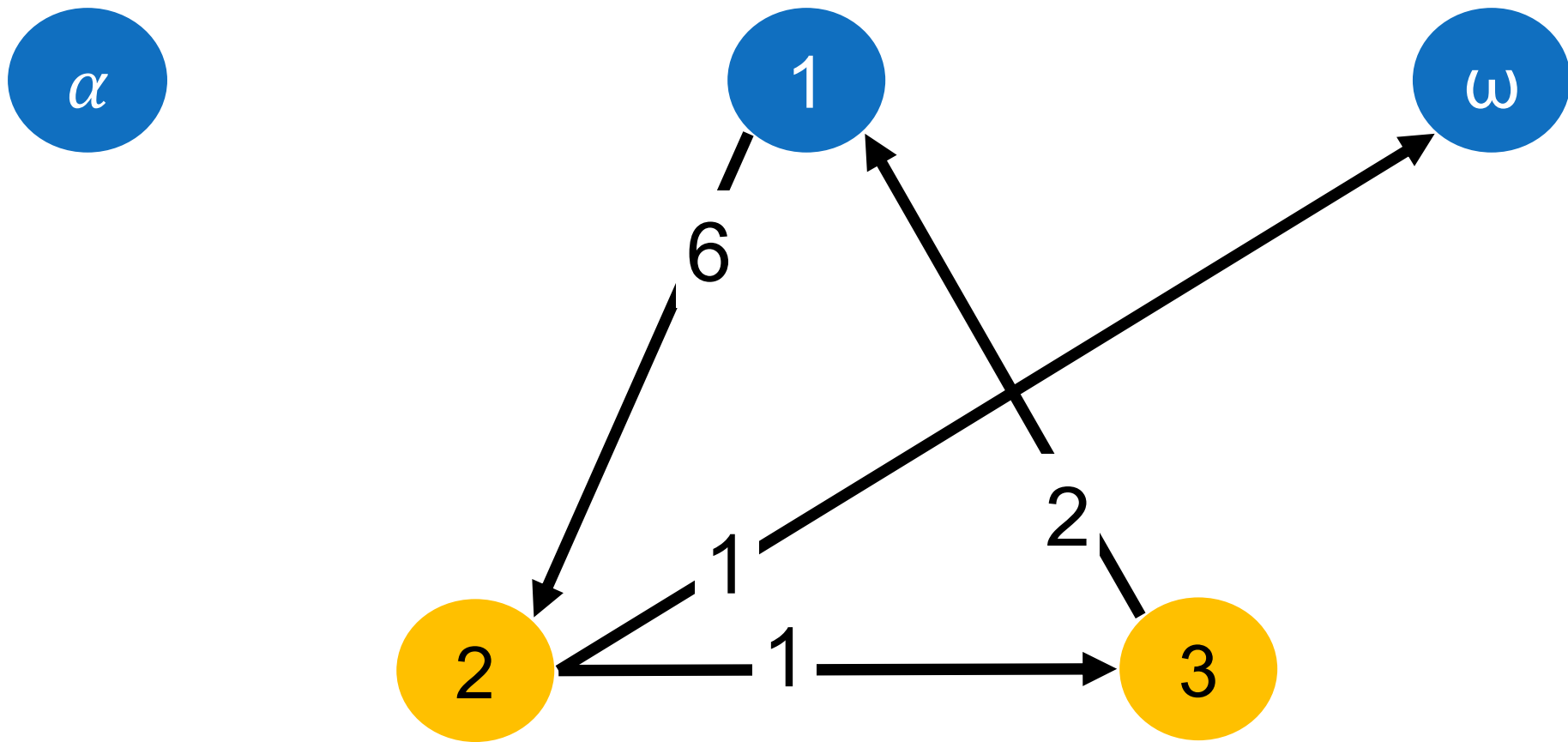
Lower bound based on the inputs

$$L_{IO} = 0 + 2 + 6 + 1 + 1 = 10$$
$$L_O < L_{IO} < \text{Dist} = 12$$



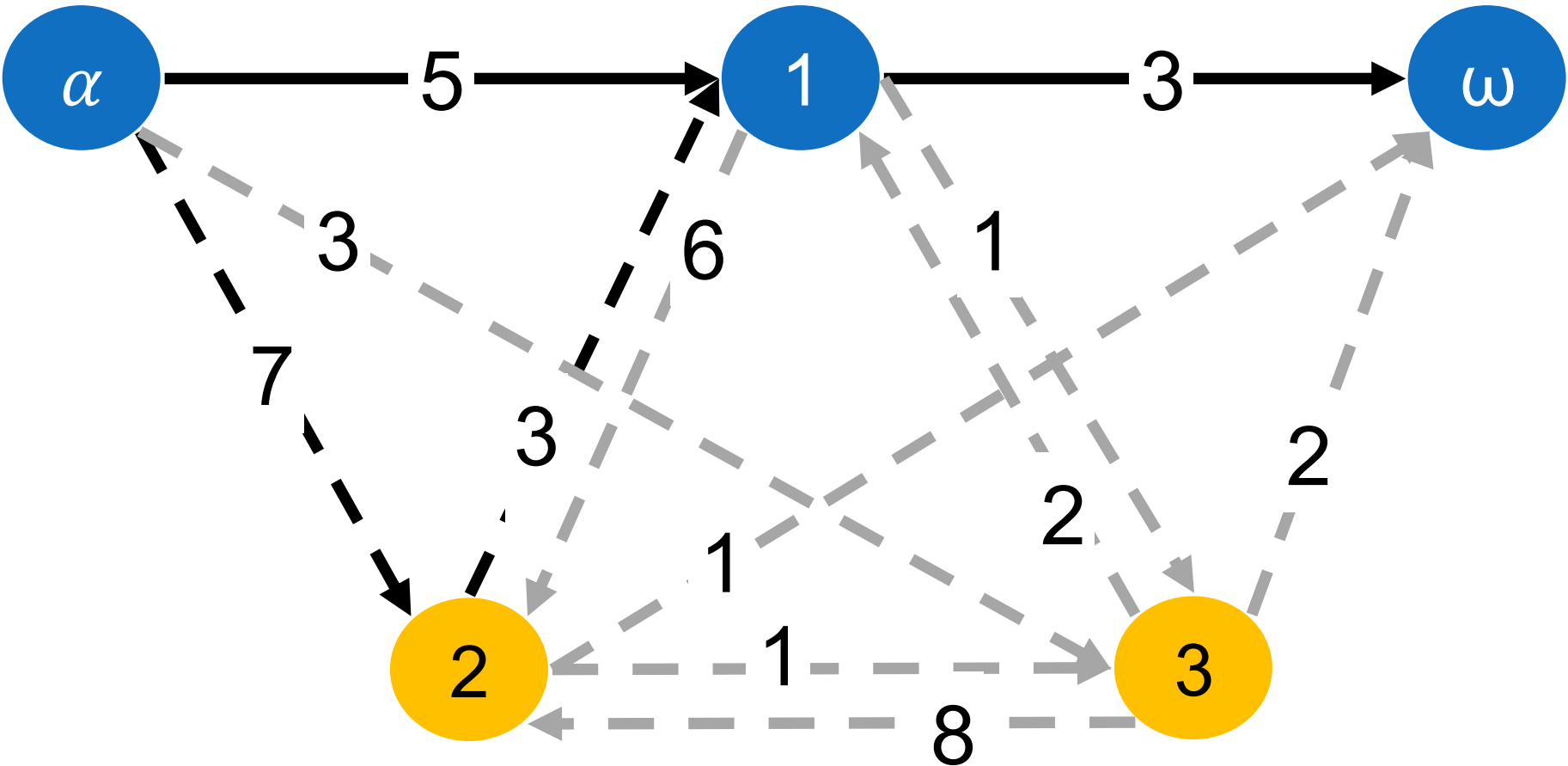
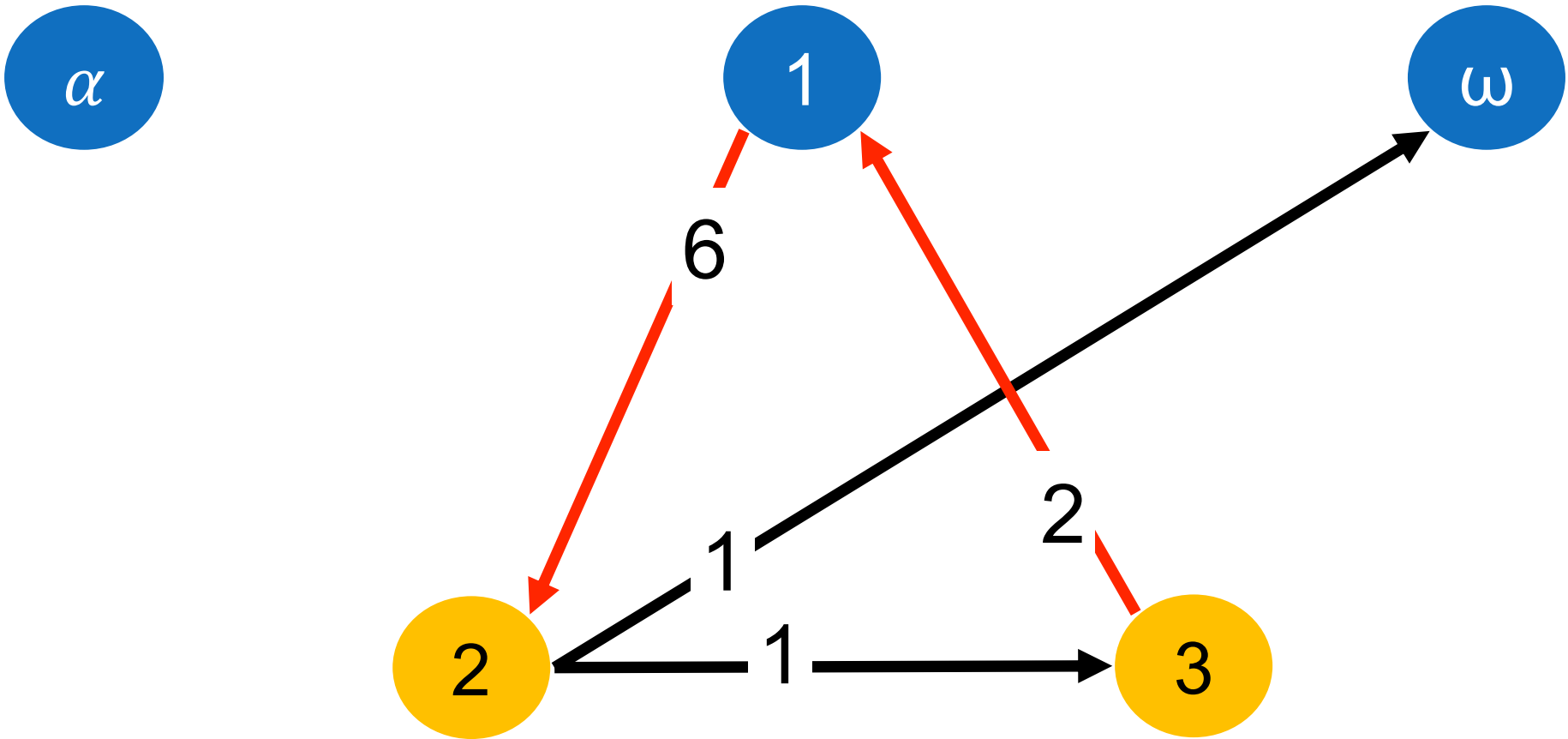
Lower bound based on the inputs

Filtering



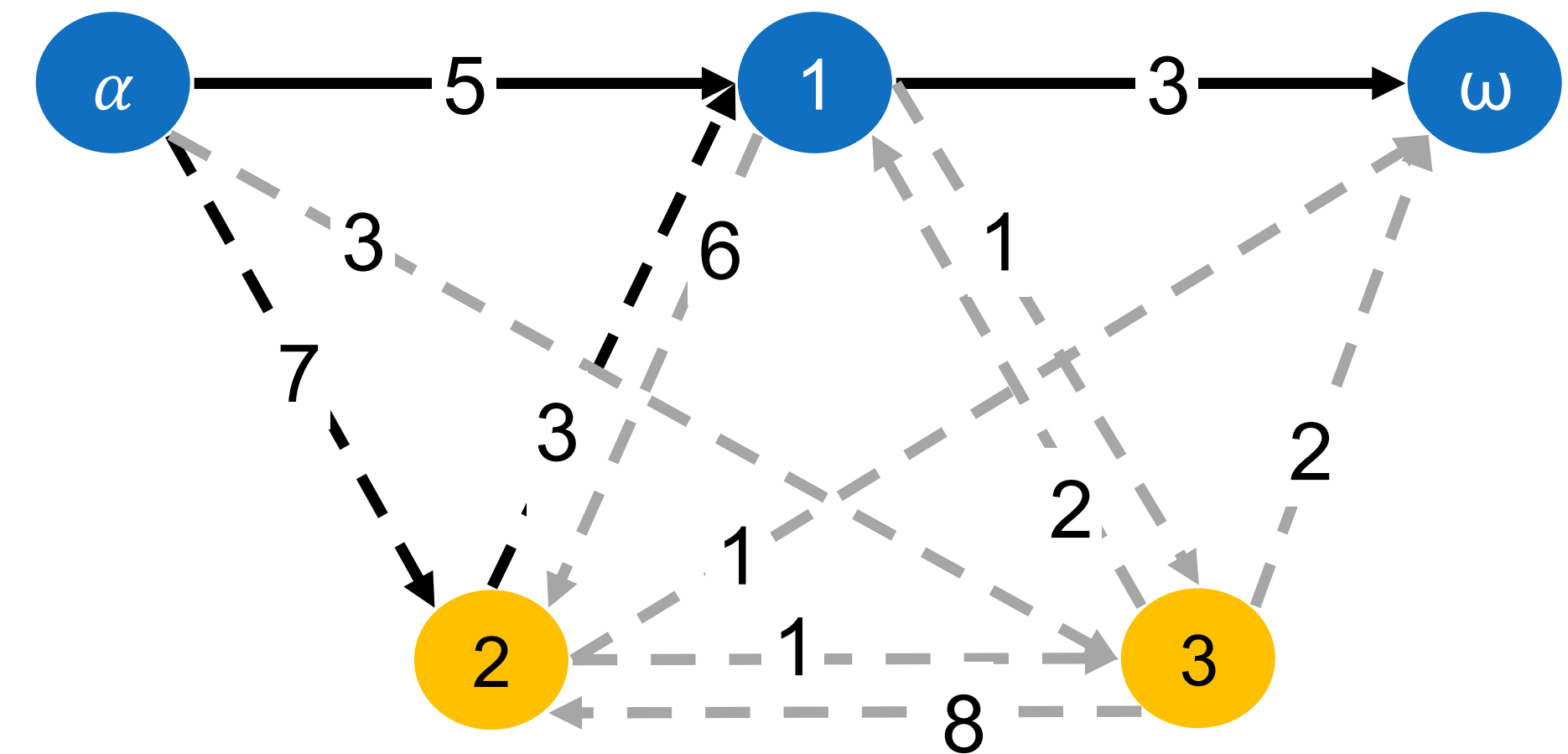
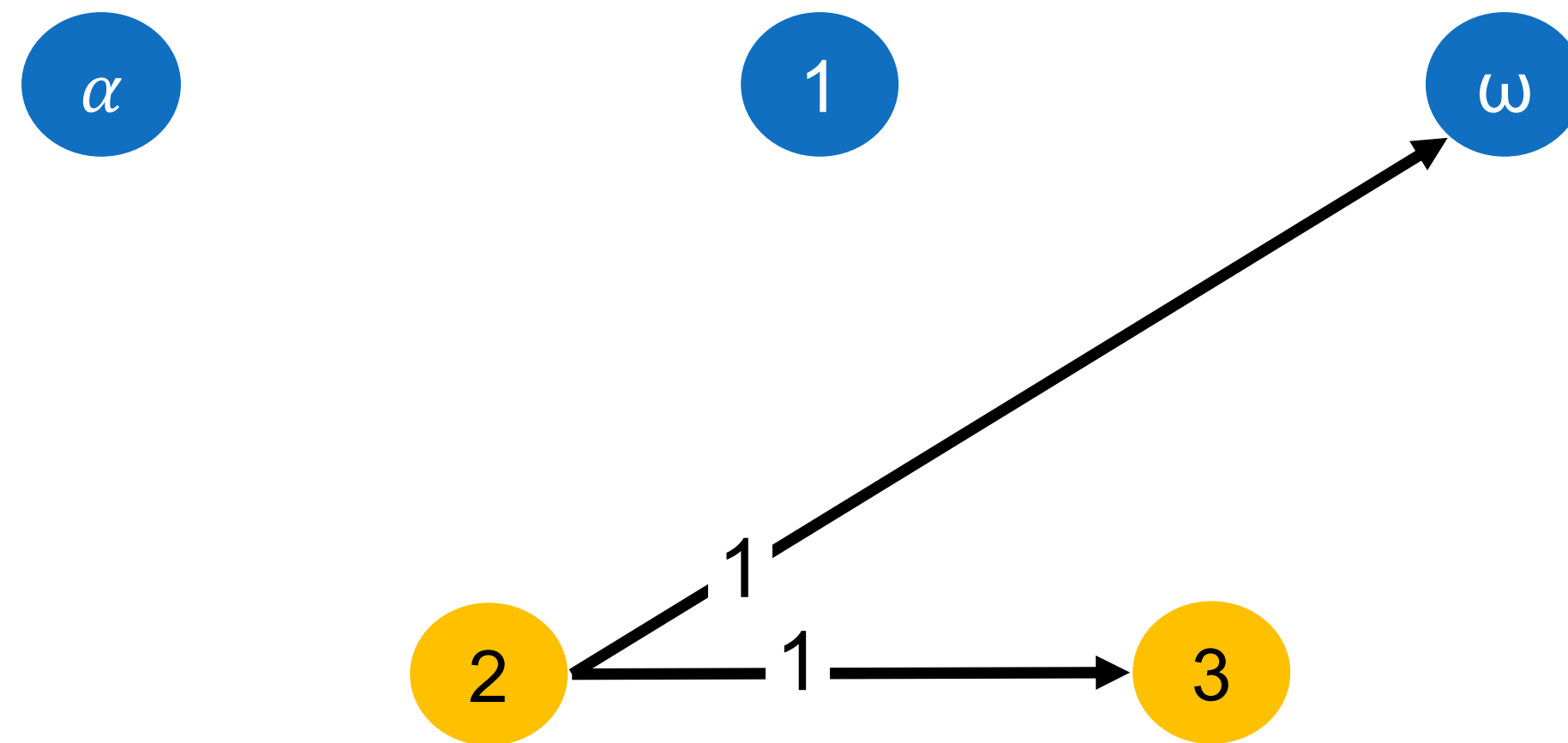
Lower bound based on the inputs

Filtering



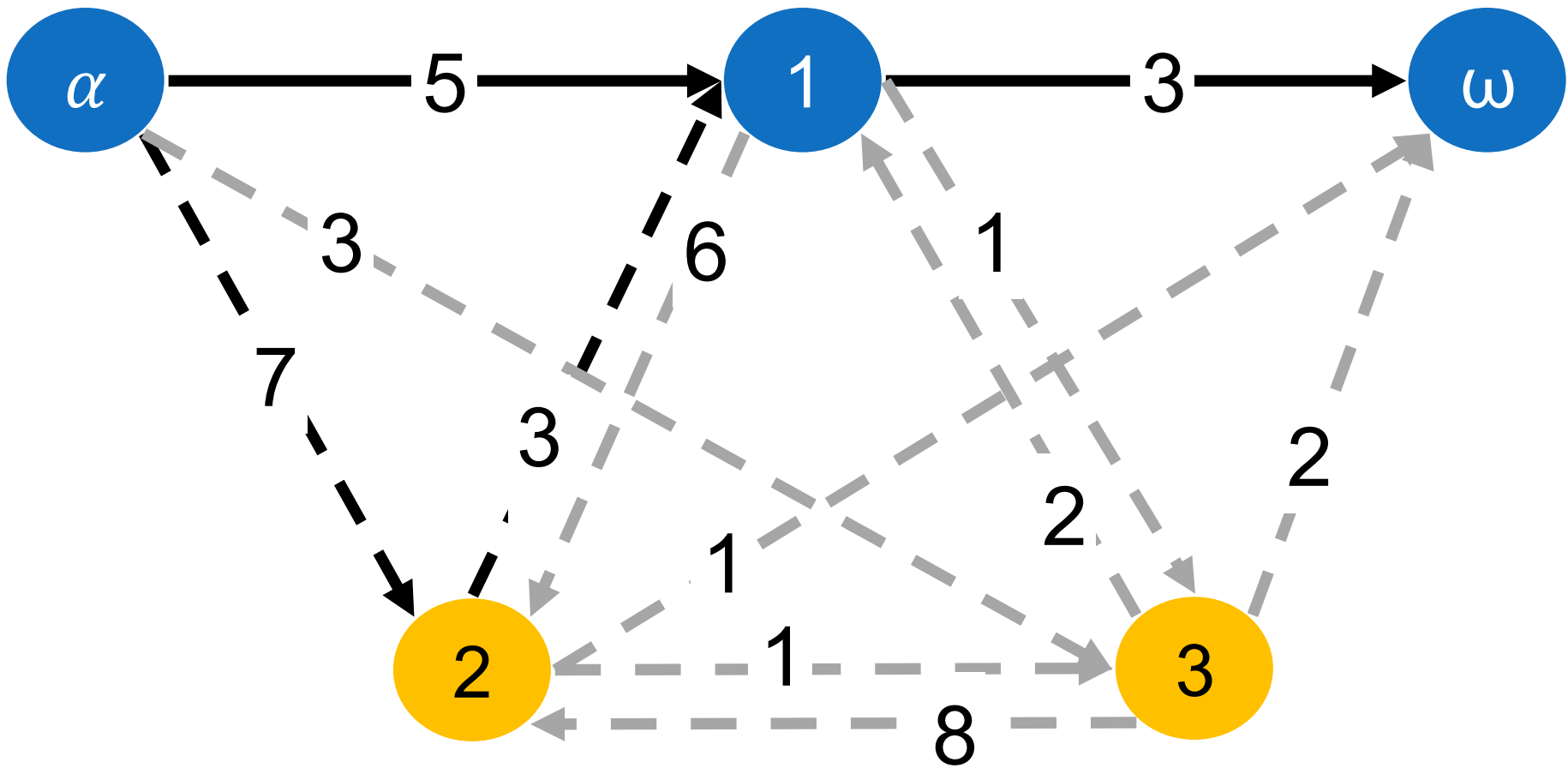
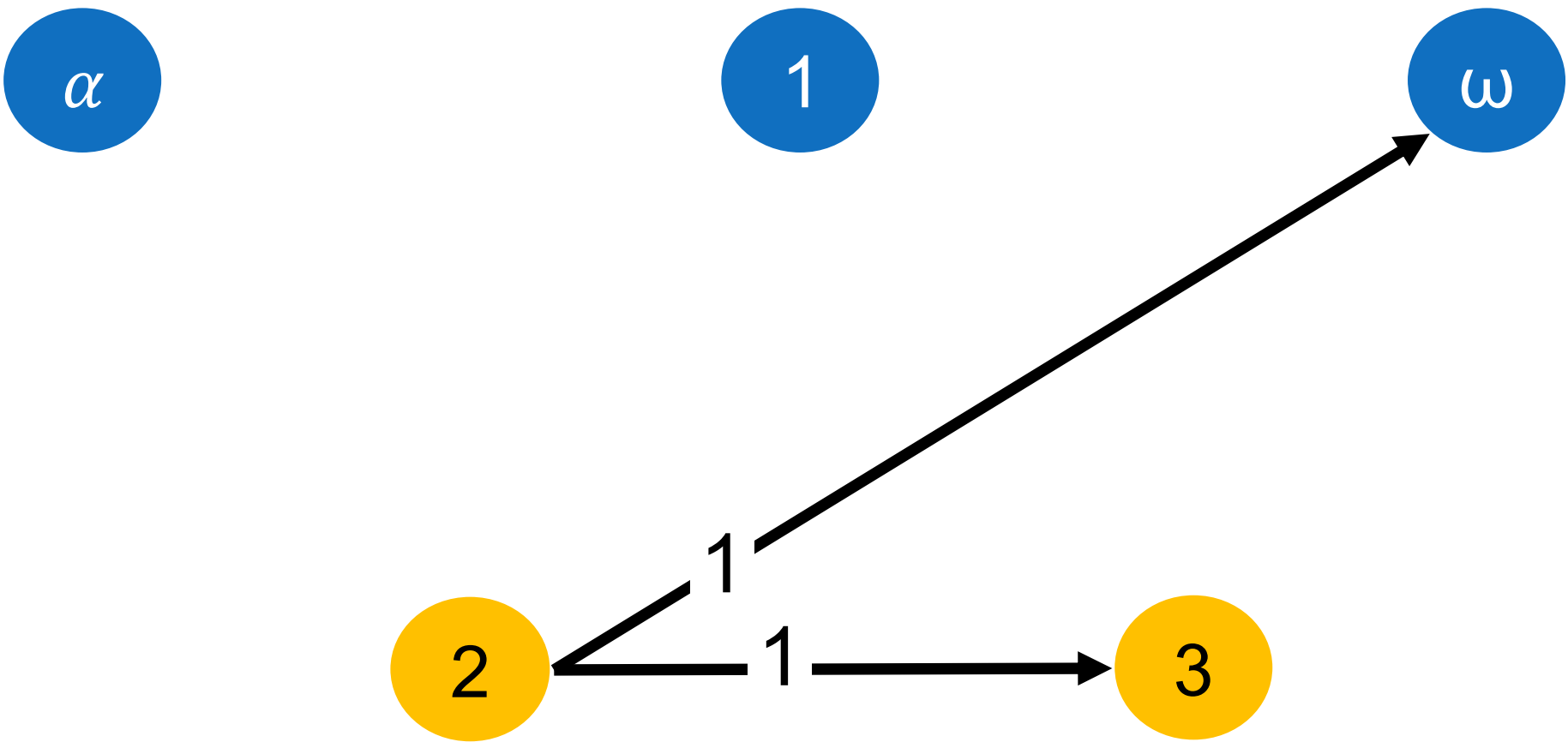
Lower bound based on the inputs

Filtering



Lower bound based on the inputs

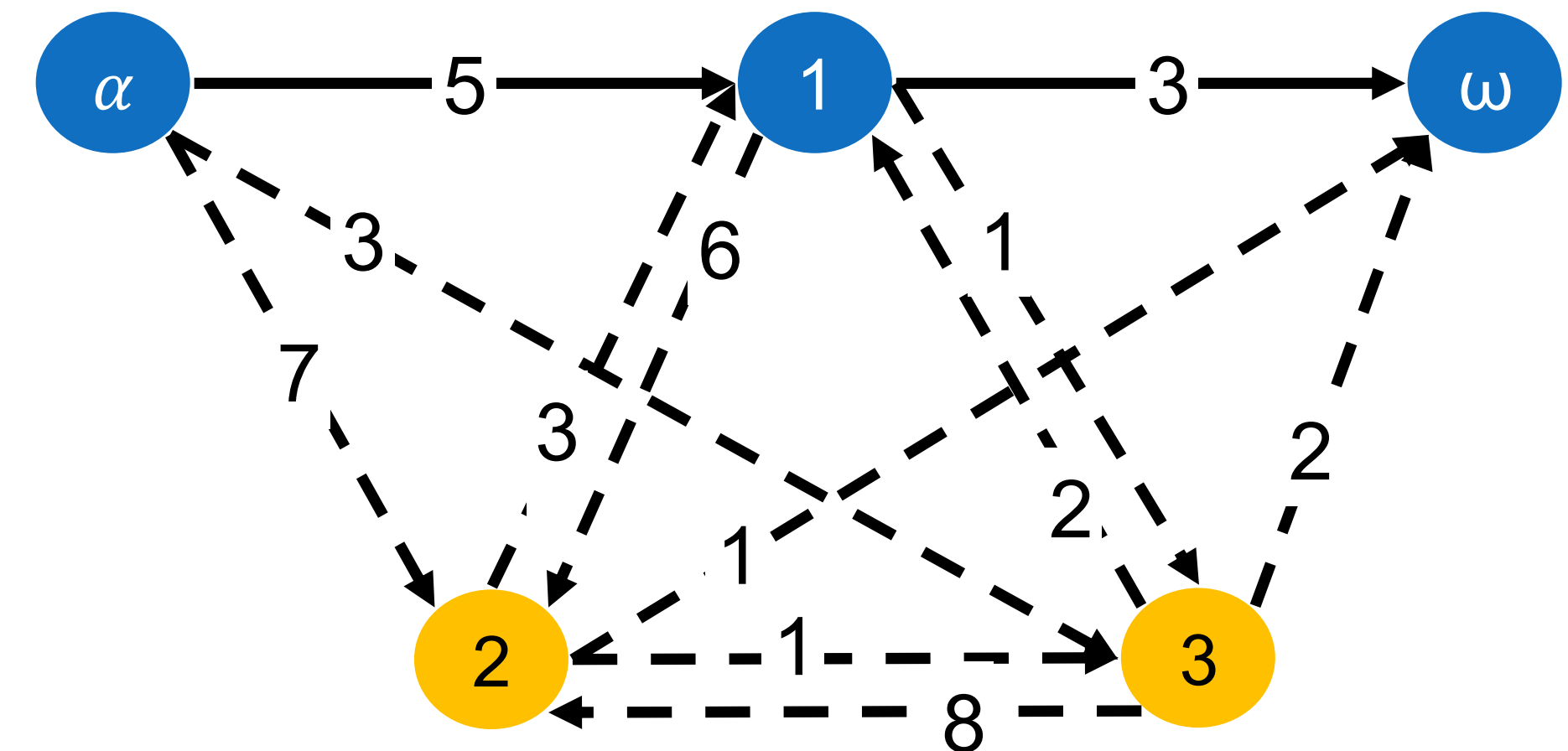
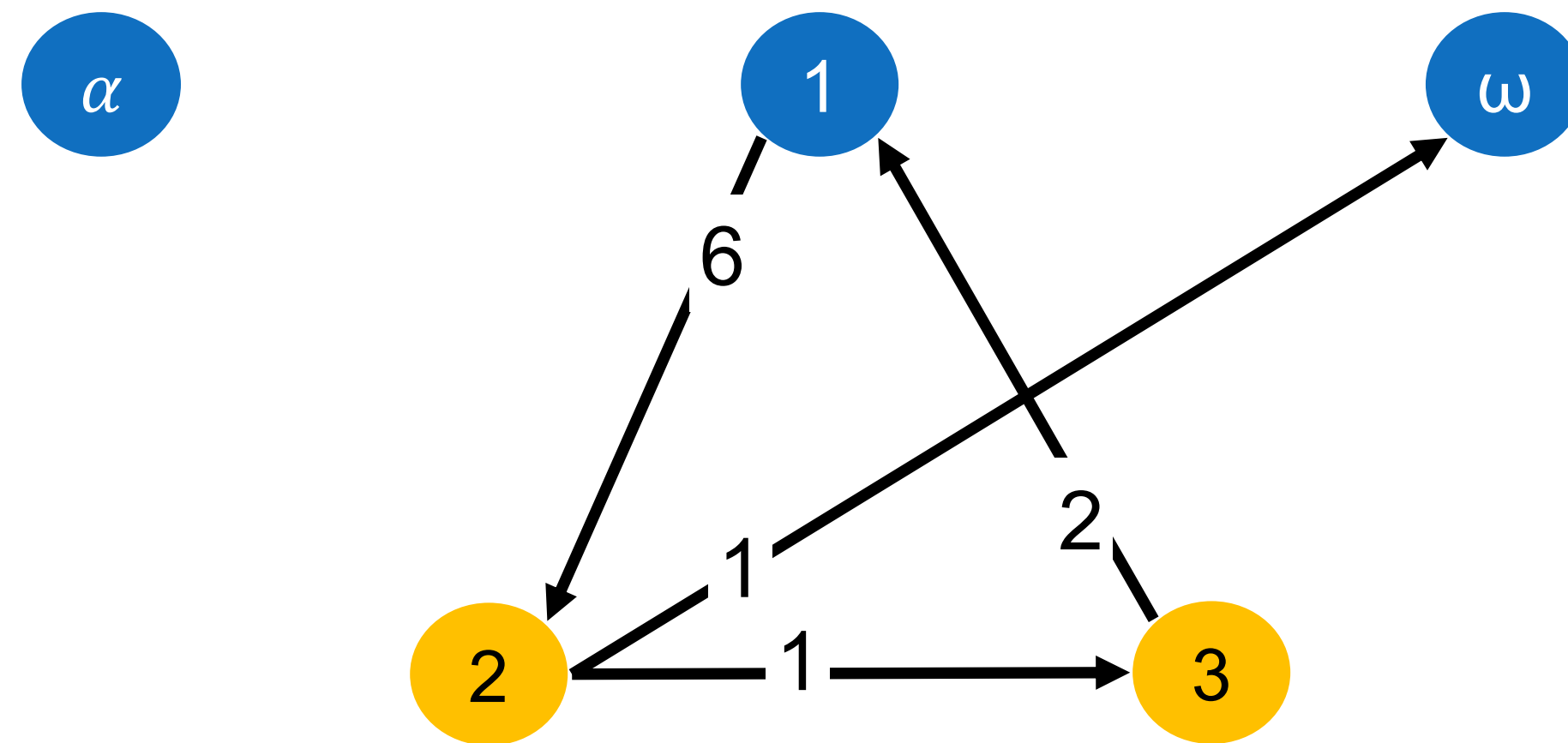
Filtering



$$L_{IO} = 1 + 1 + \Delta(\alpha, 2, 1) = 7$$
$$L_{IO} < \text{Dist} = 12$$

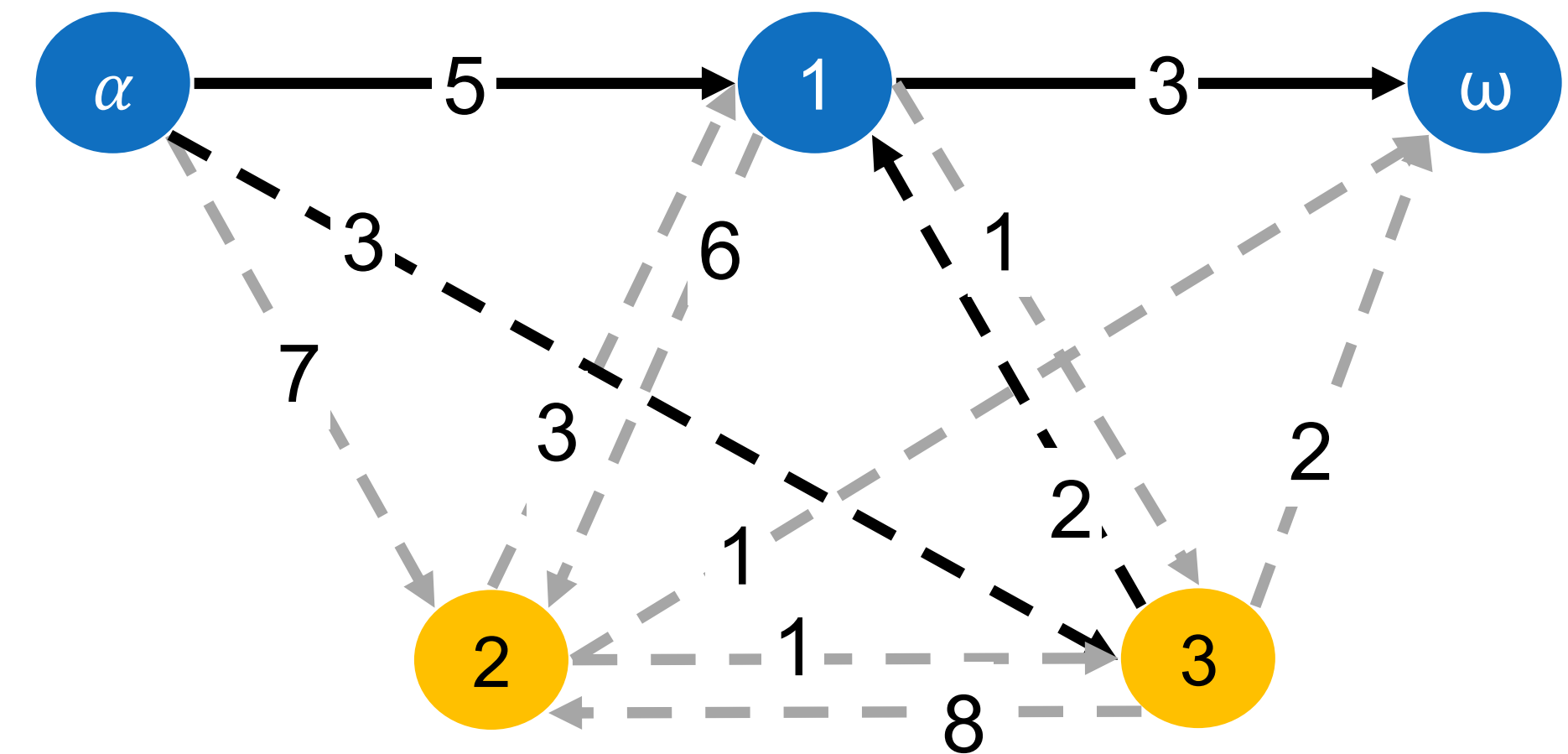
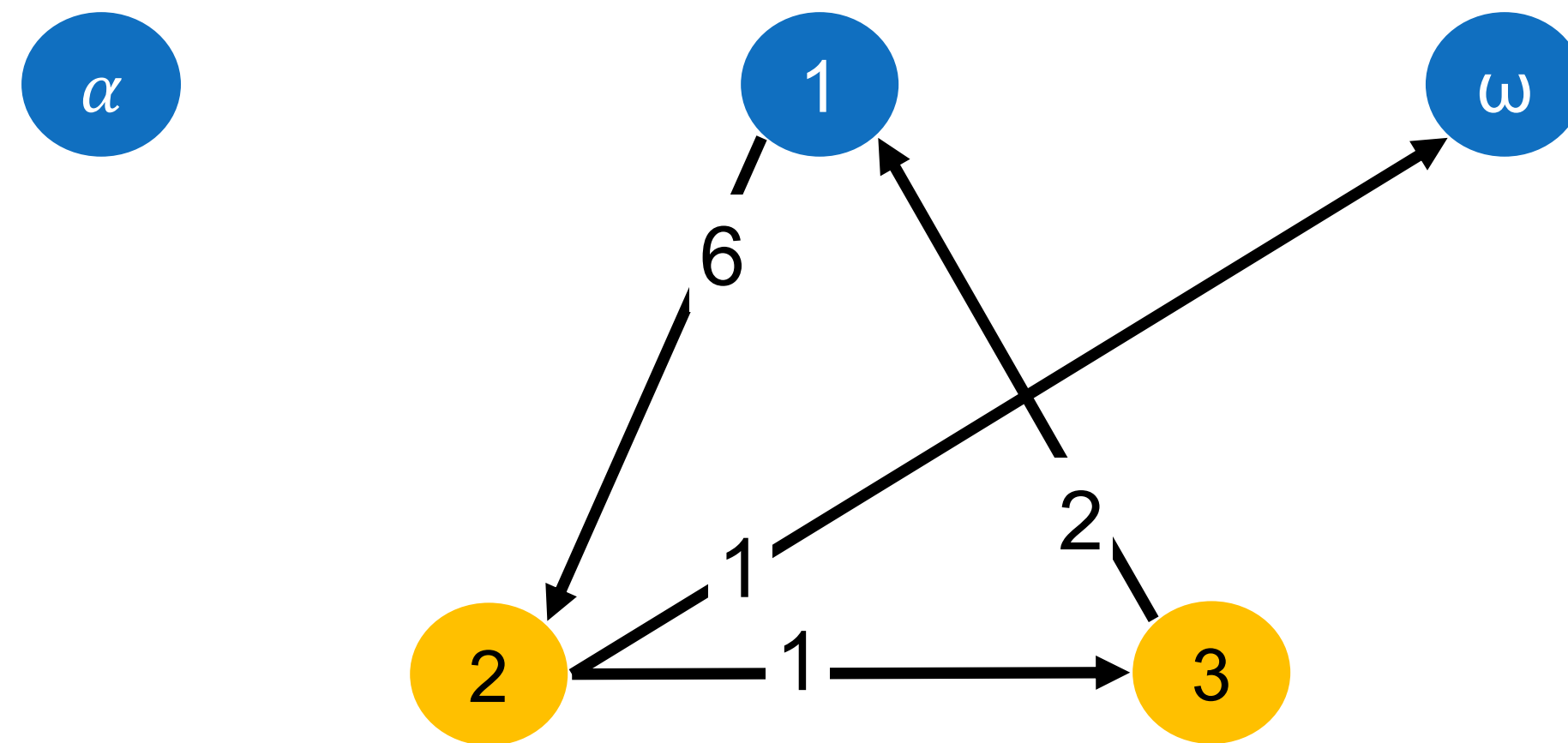
Lower bound based on the inputs

Filtering



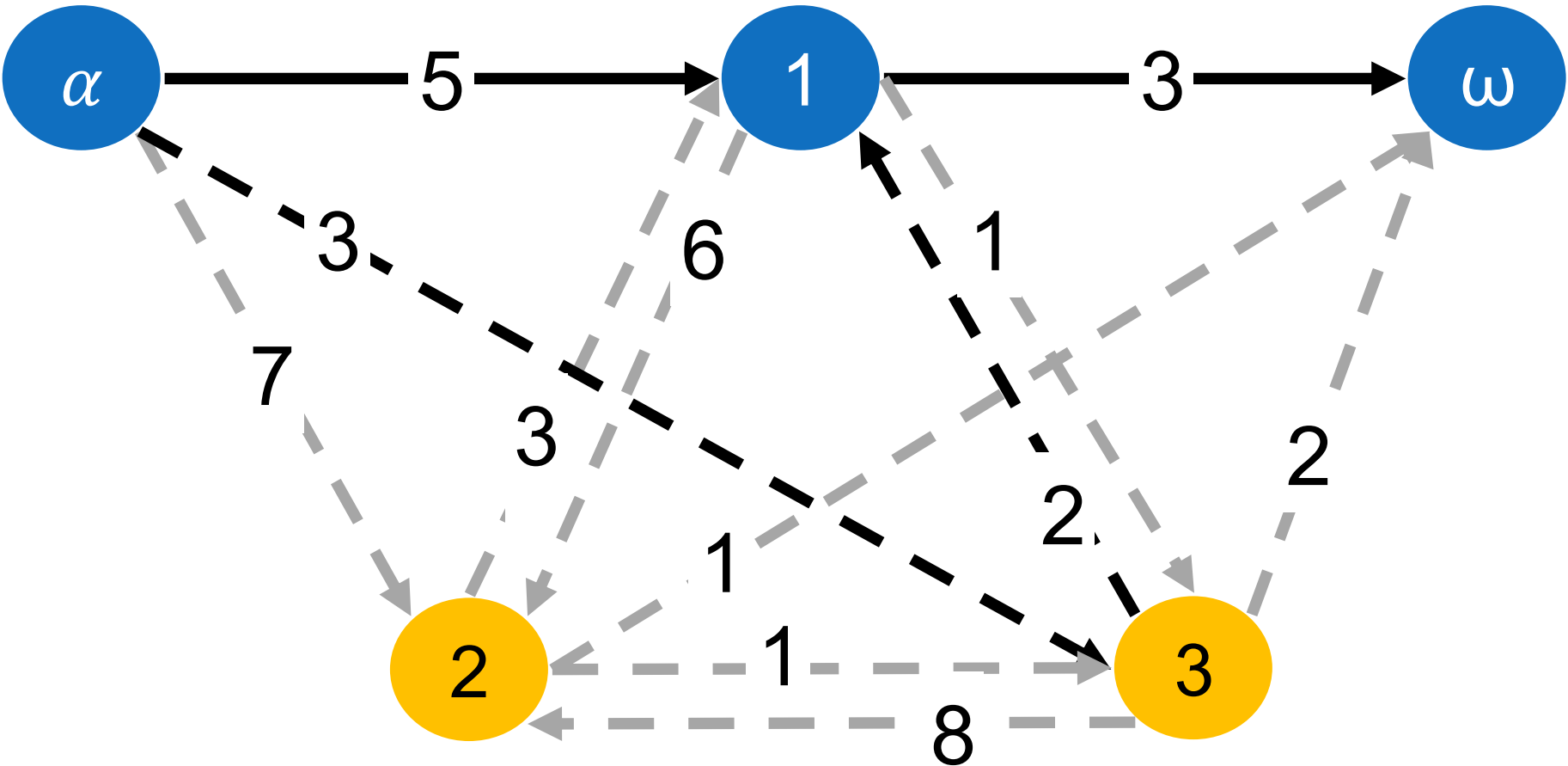
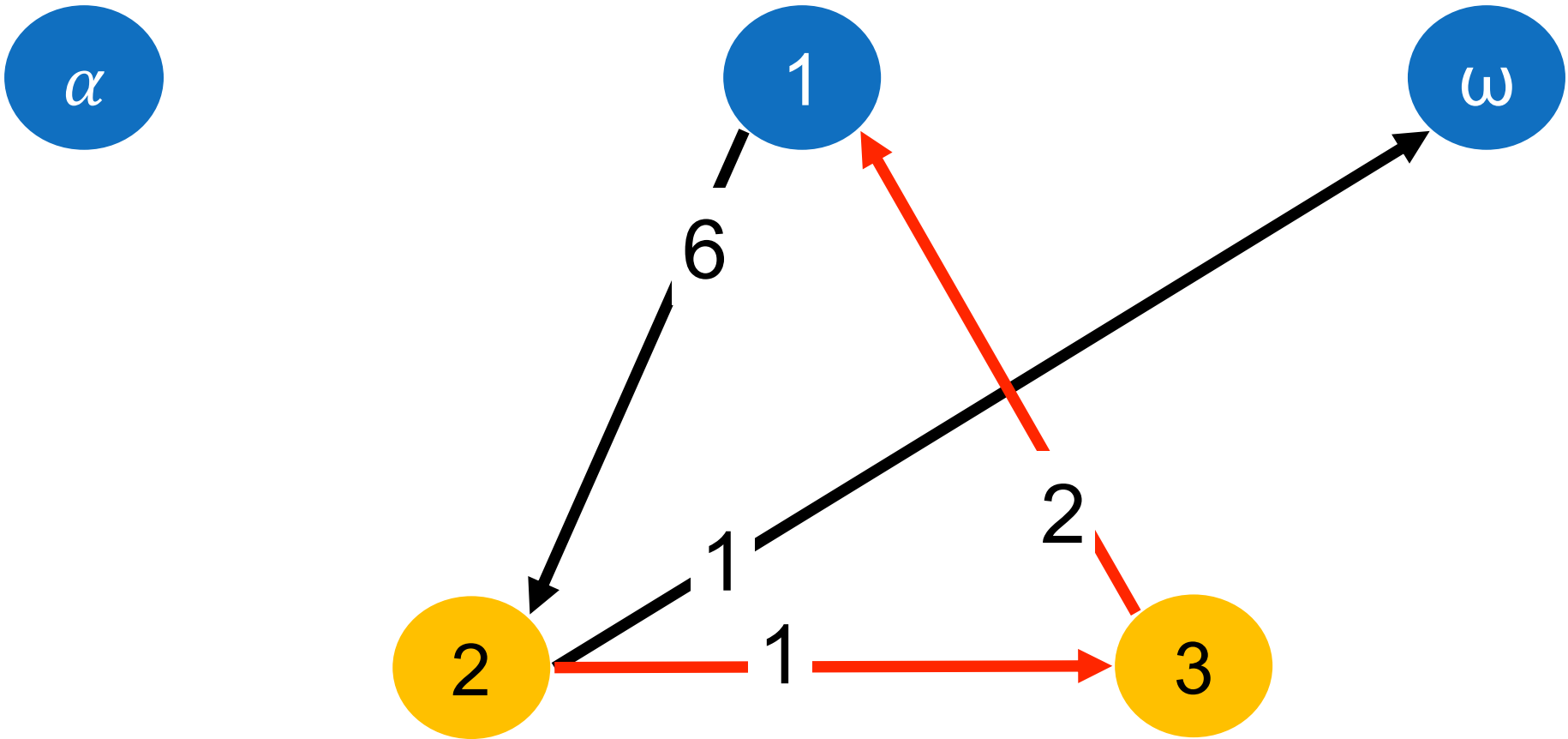
Lower bound based on the inputs

Filtering



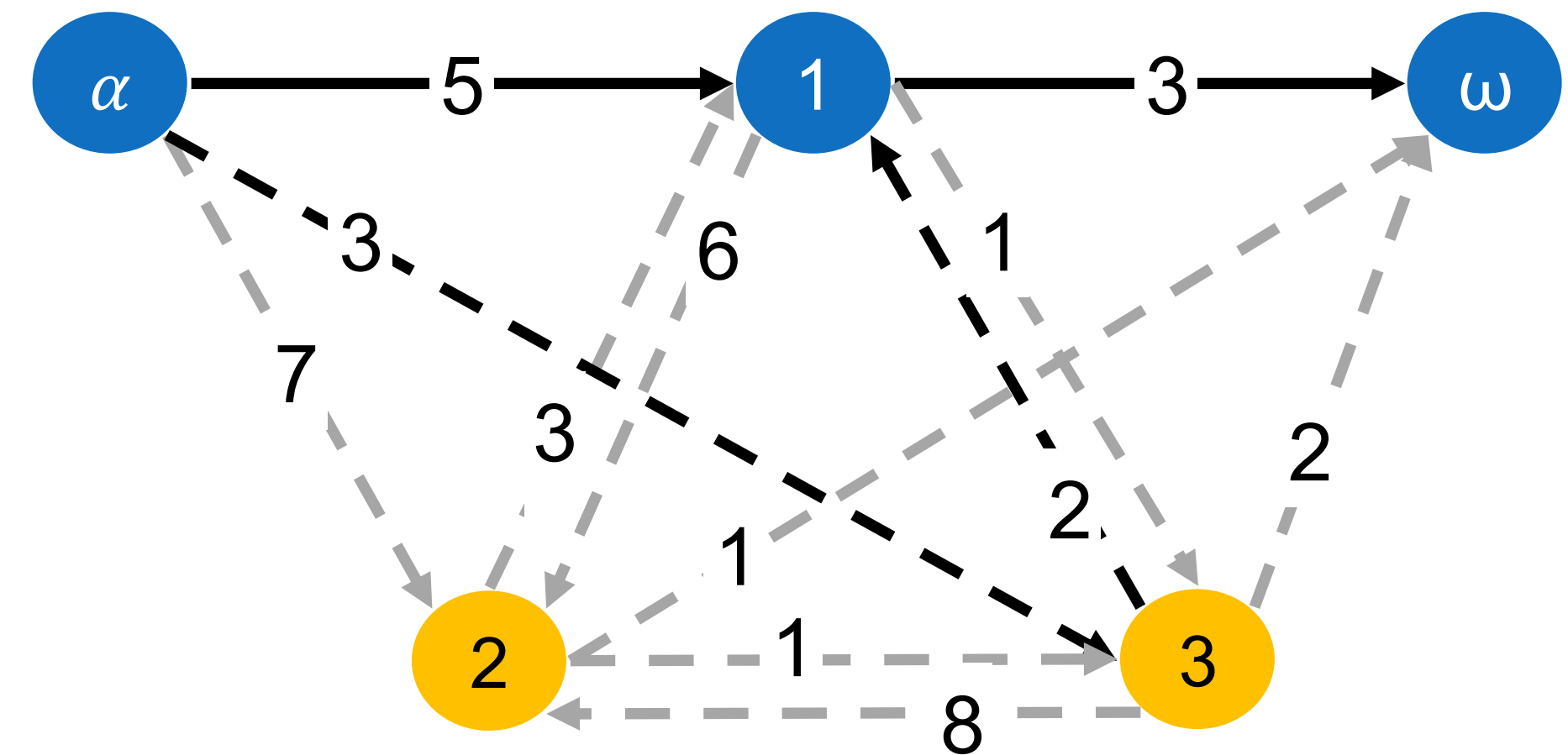
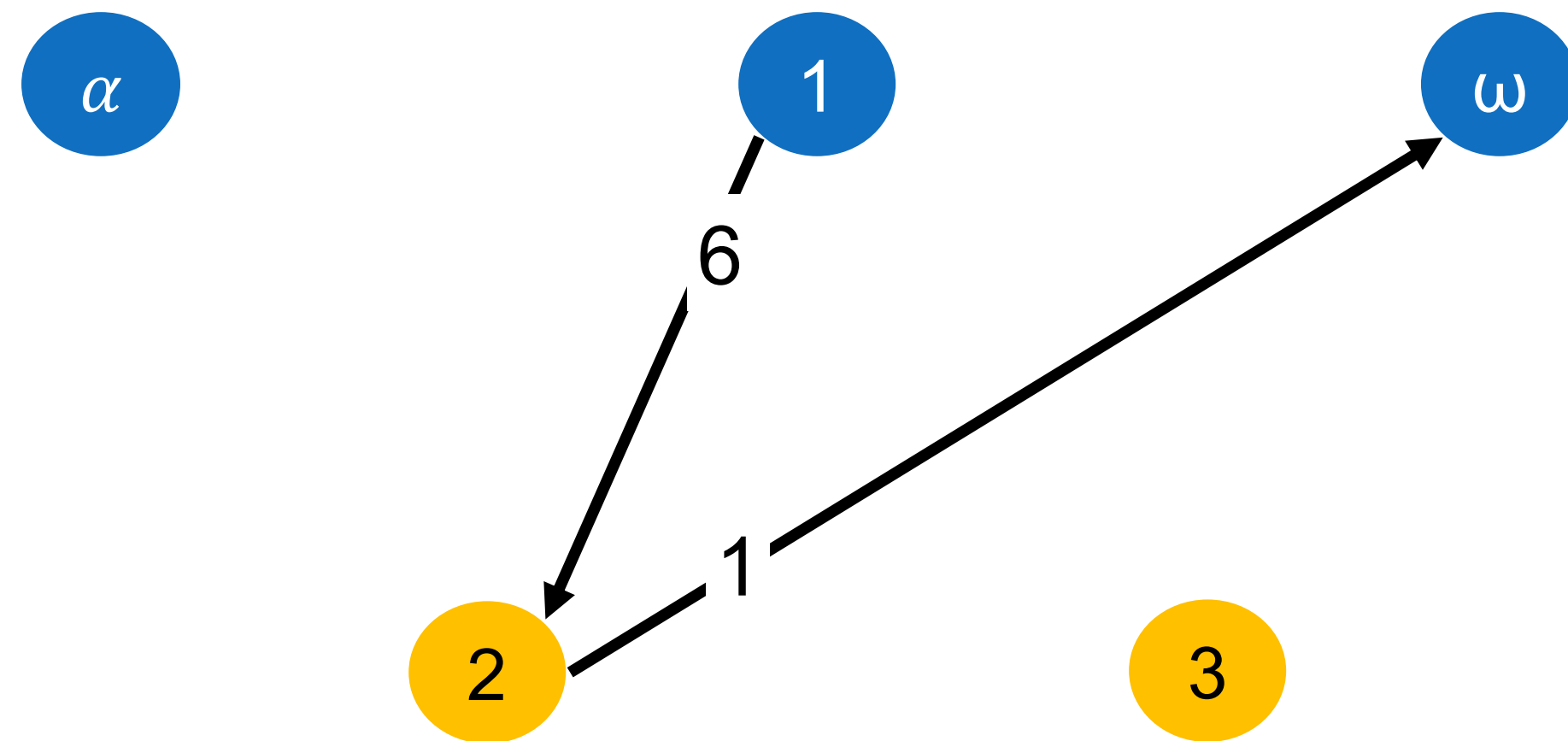
Lower bound based on the inputs

Filtering



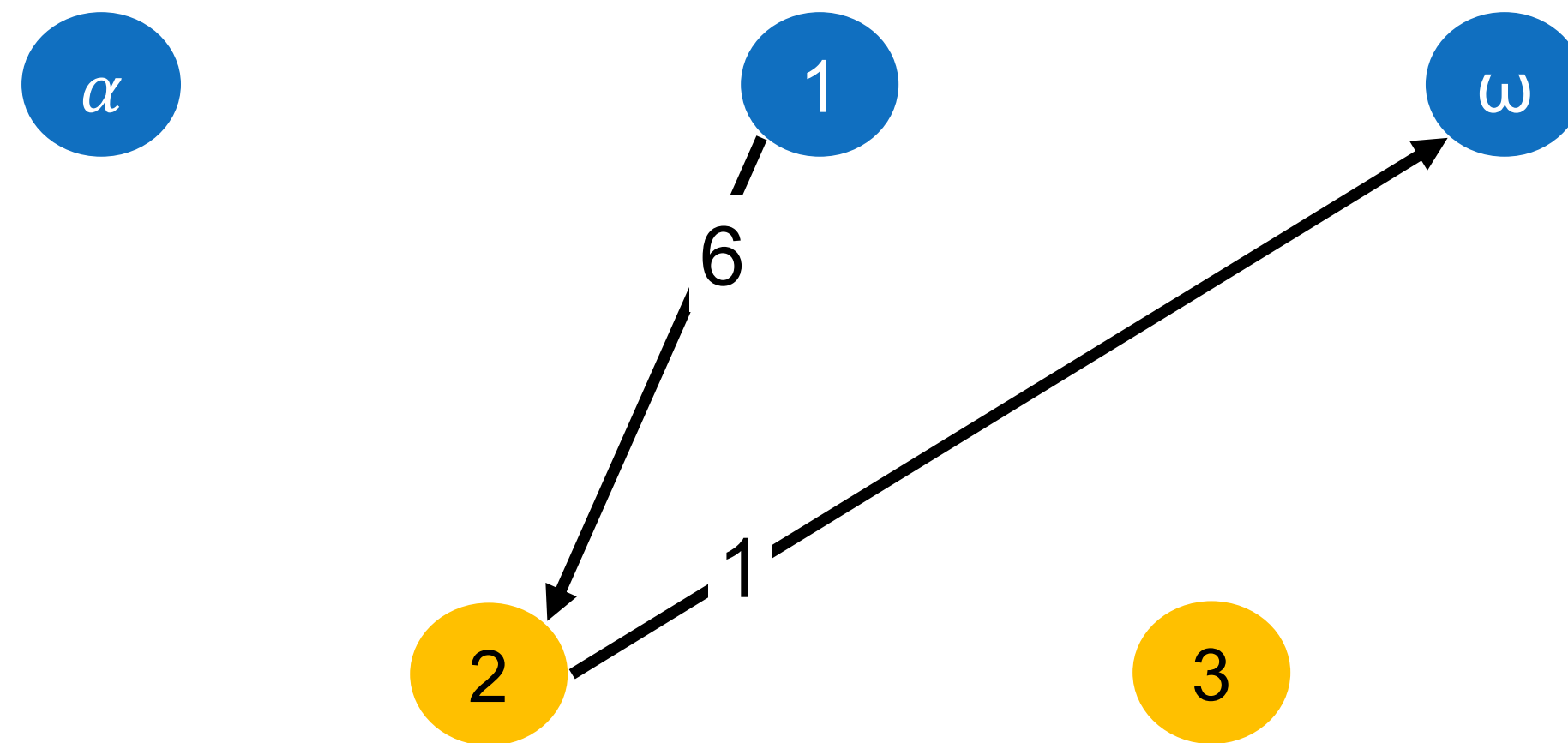
Lower bound based on the inputs

Filtering

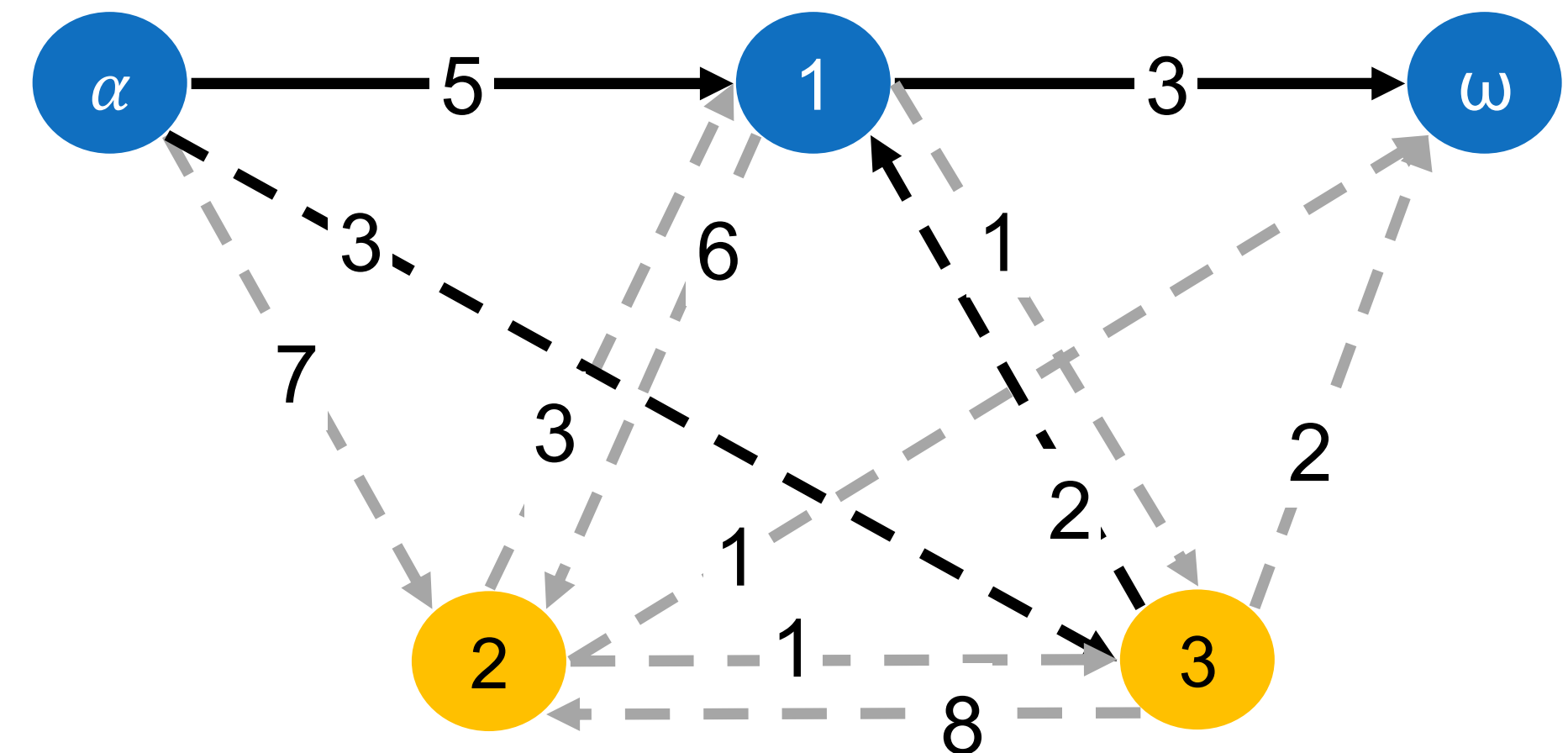


Lower bound based on the inputs

Filtering



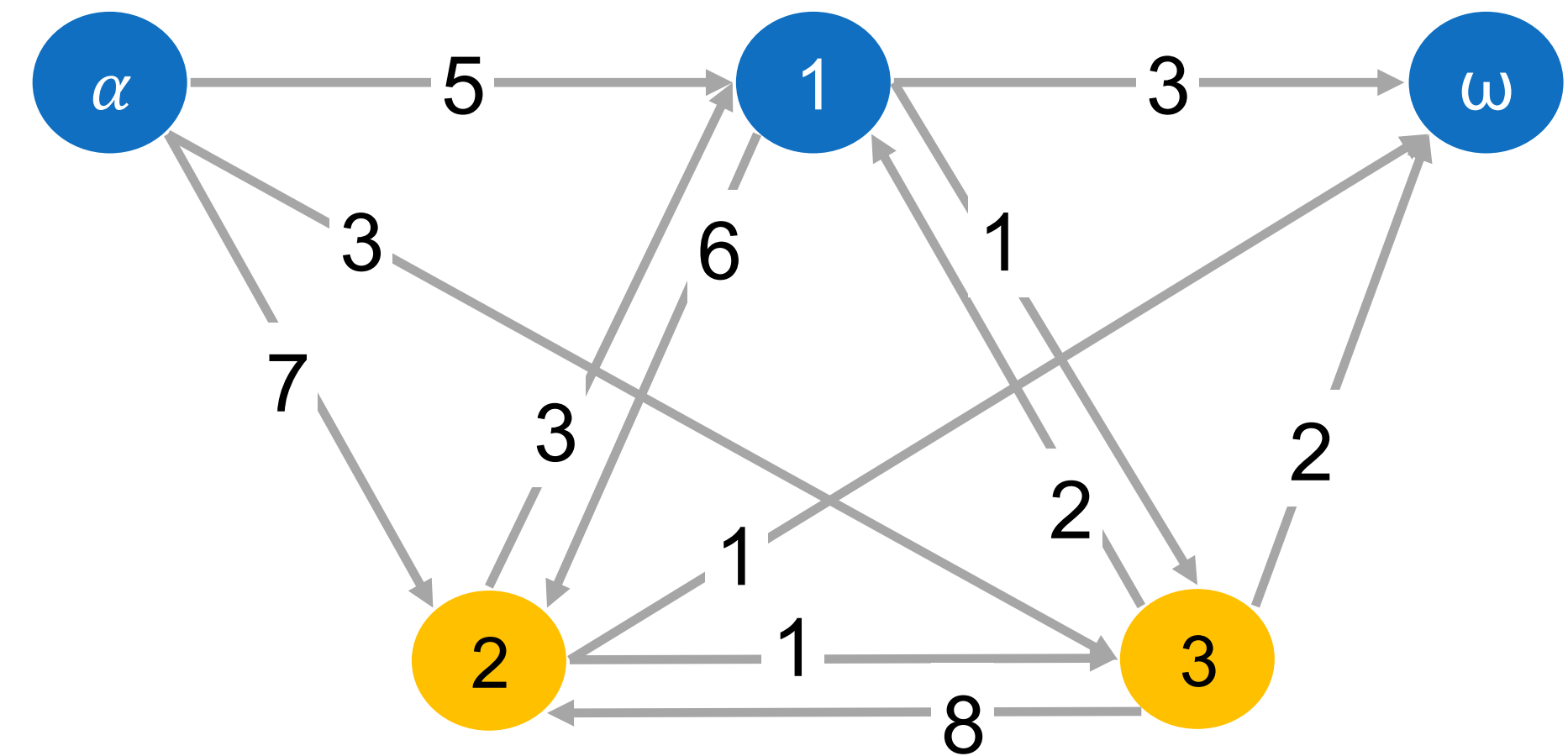
$$L_{IO} = 6 + 1 + \Delta(\alpha, 3, 1) = 7$$
$$L_{IO} < \text{Dist} = 12$$



Lower Bound Based on Detours

Lower Bound Based on Detours

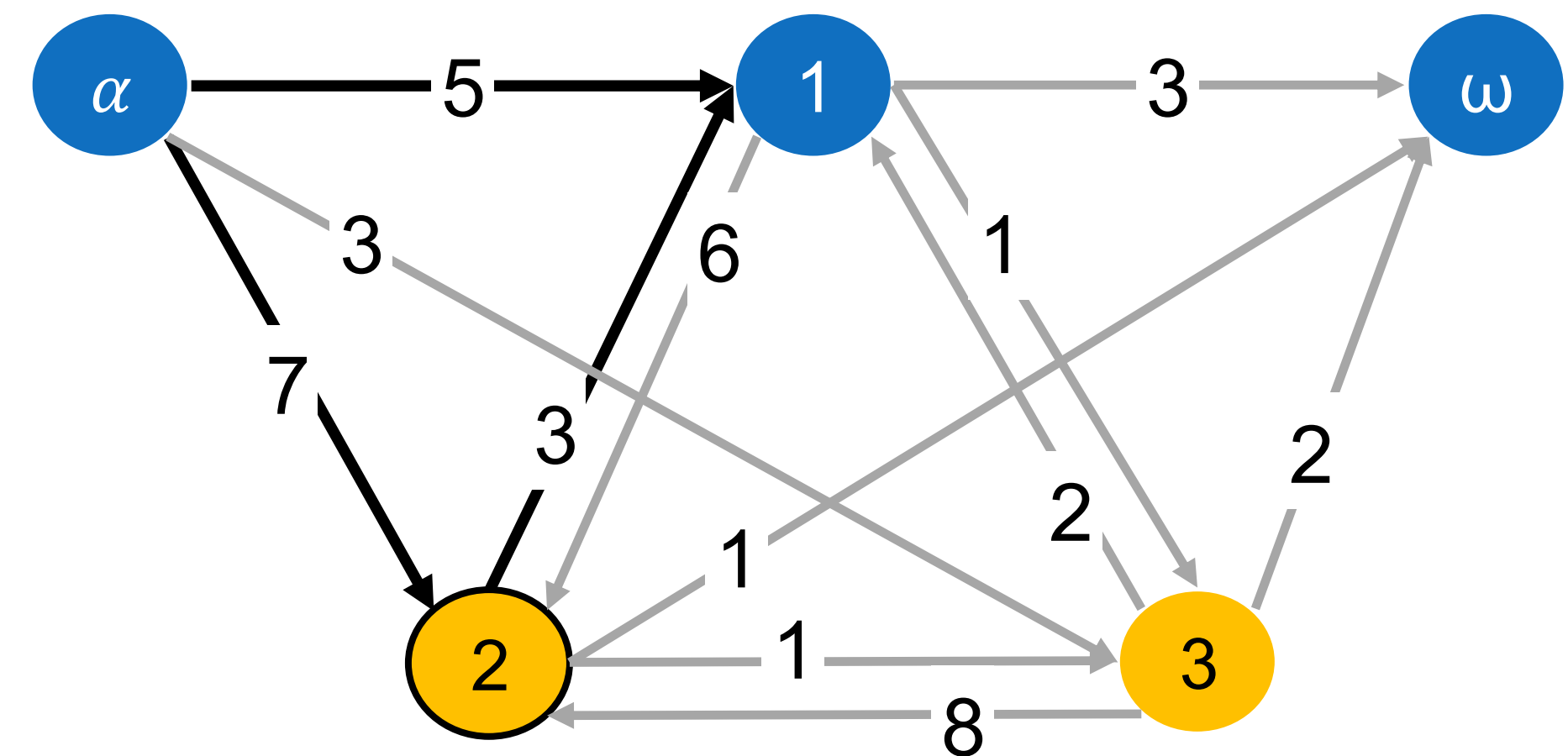
$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$



Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\Delta(\alpha, 2, 1) = 5$$

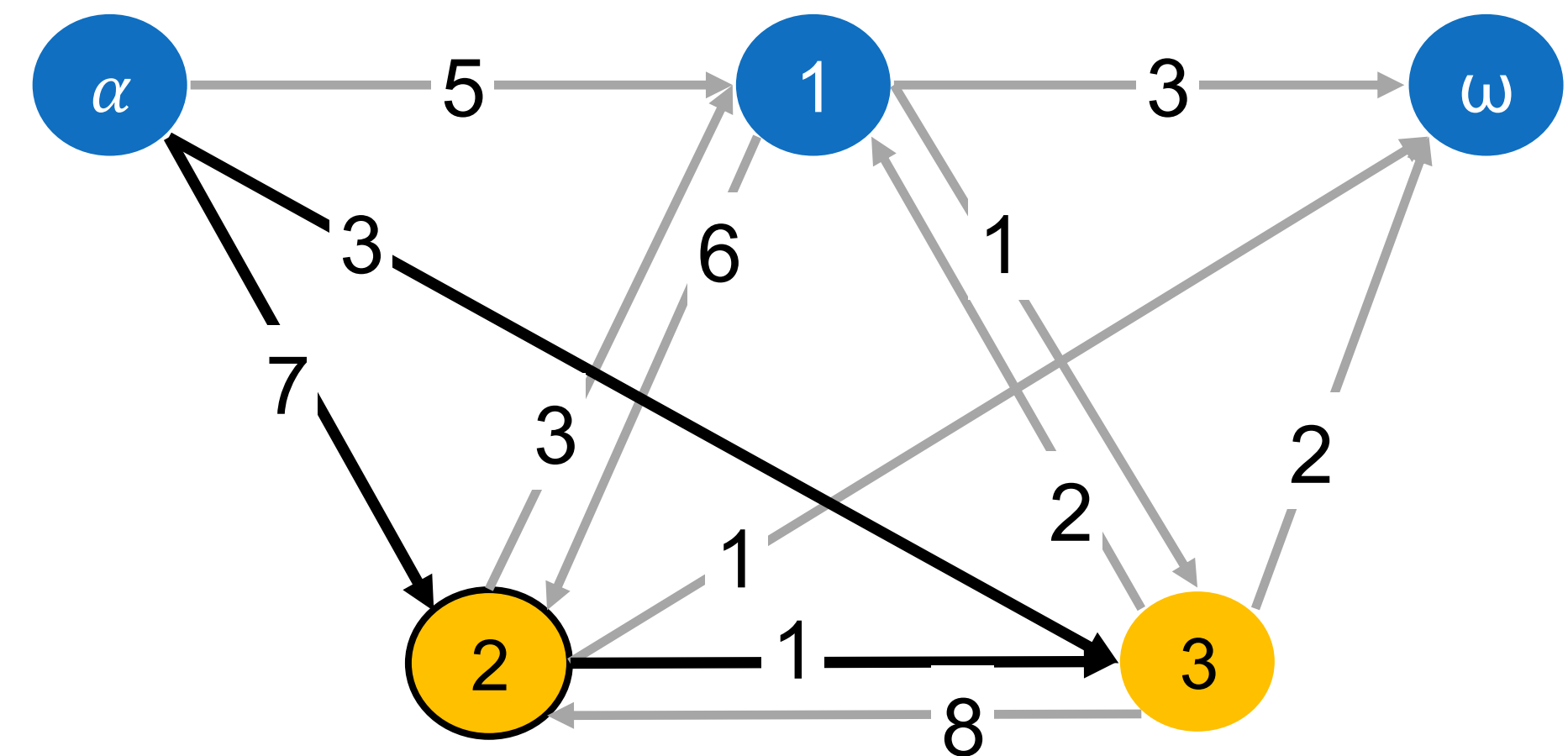


Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\Delta(\alpha, 2, 1) = 5$$

$$\Delta(\alpha, 2, 3) = 5$$



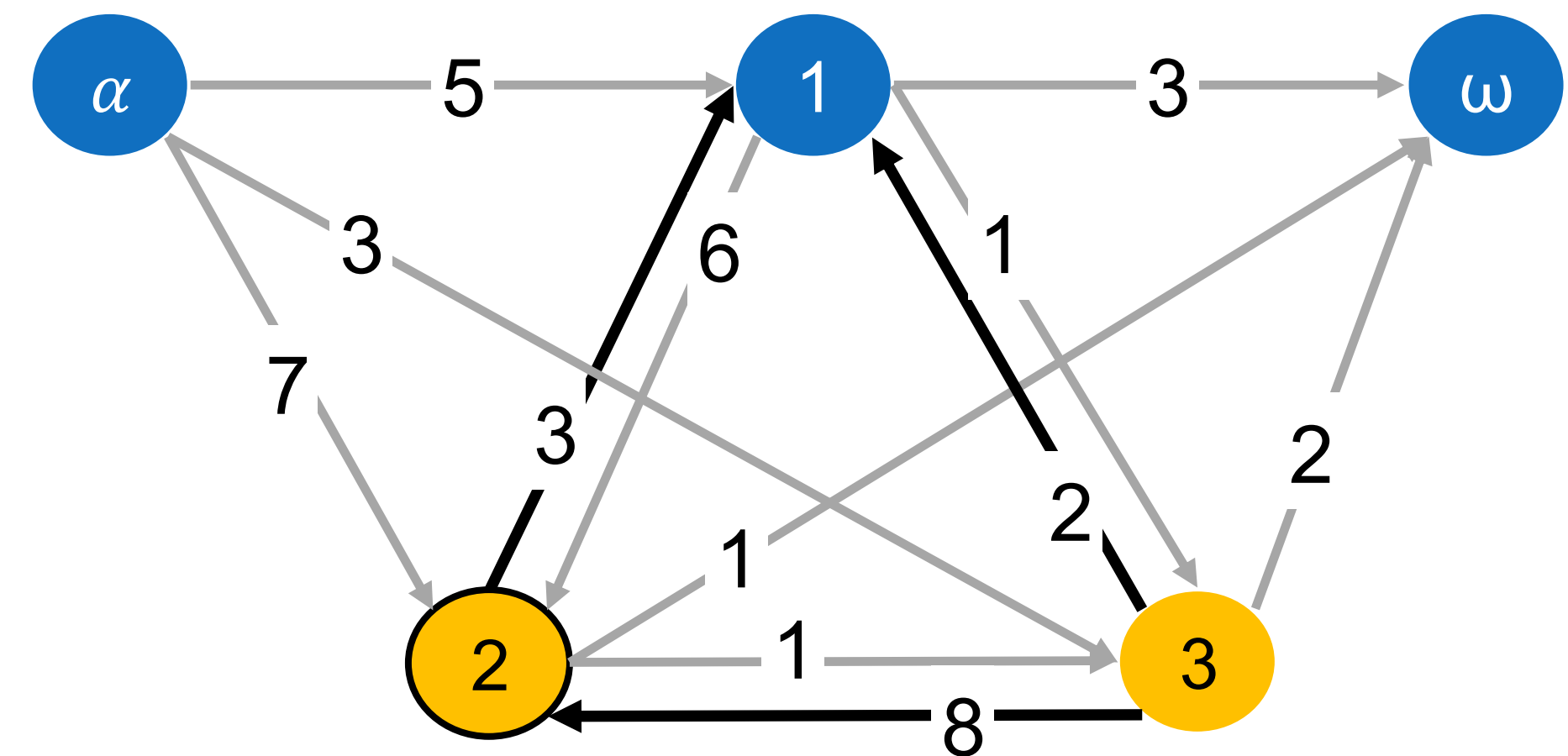
Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\Delta(\alpha, 2, 1) = 5$$

$$\Delta(\alpha, 2, 3) = 5$$

$$\Delta(3, 2, 1) = 9$$



Lower Bound Based on Detours

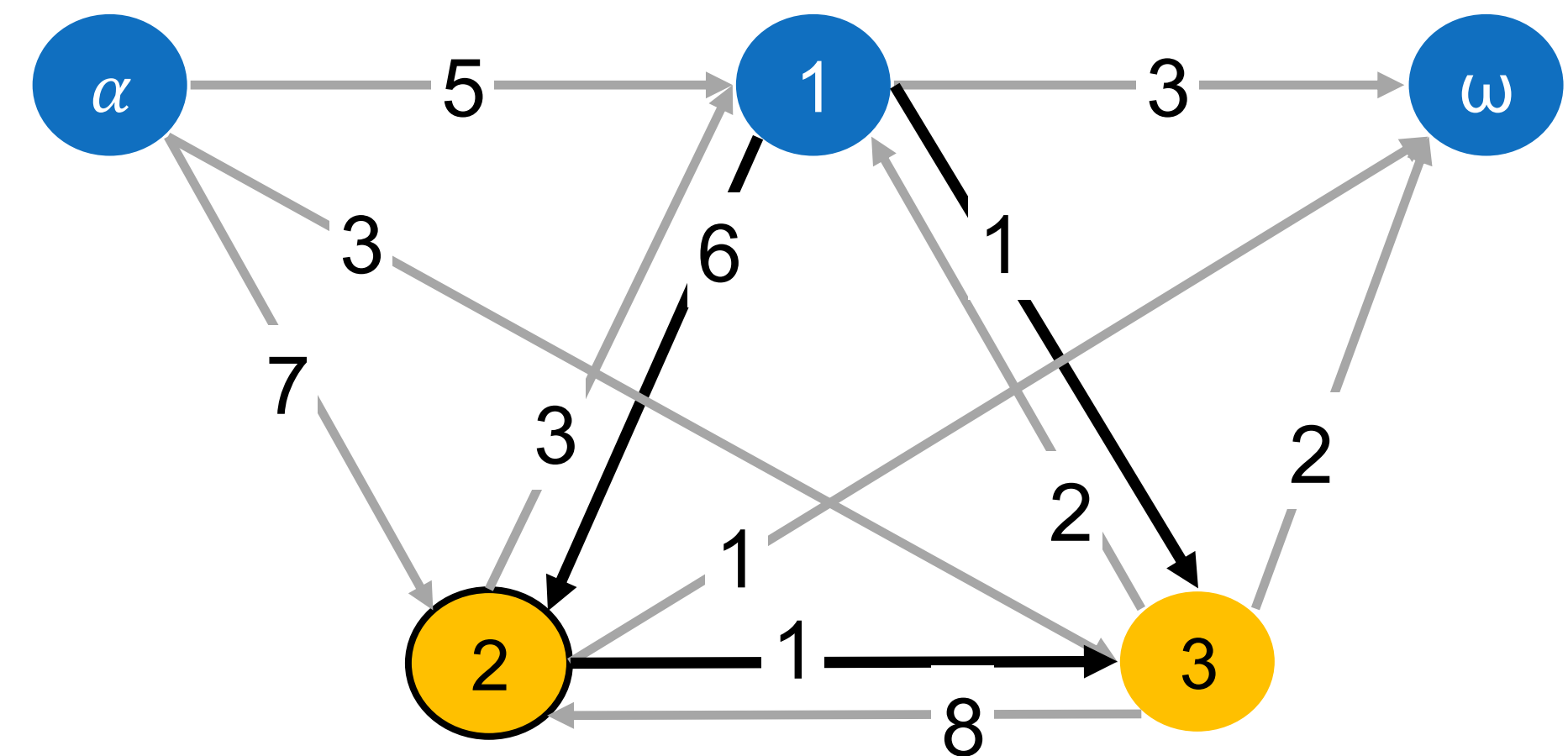
$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\Delta(\alpha, 2, 1) = 5$$

$$\Delta(\alpha, 2, 3) = 5$$

$$\Delta(3, 2, 1) = 9$$

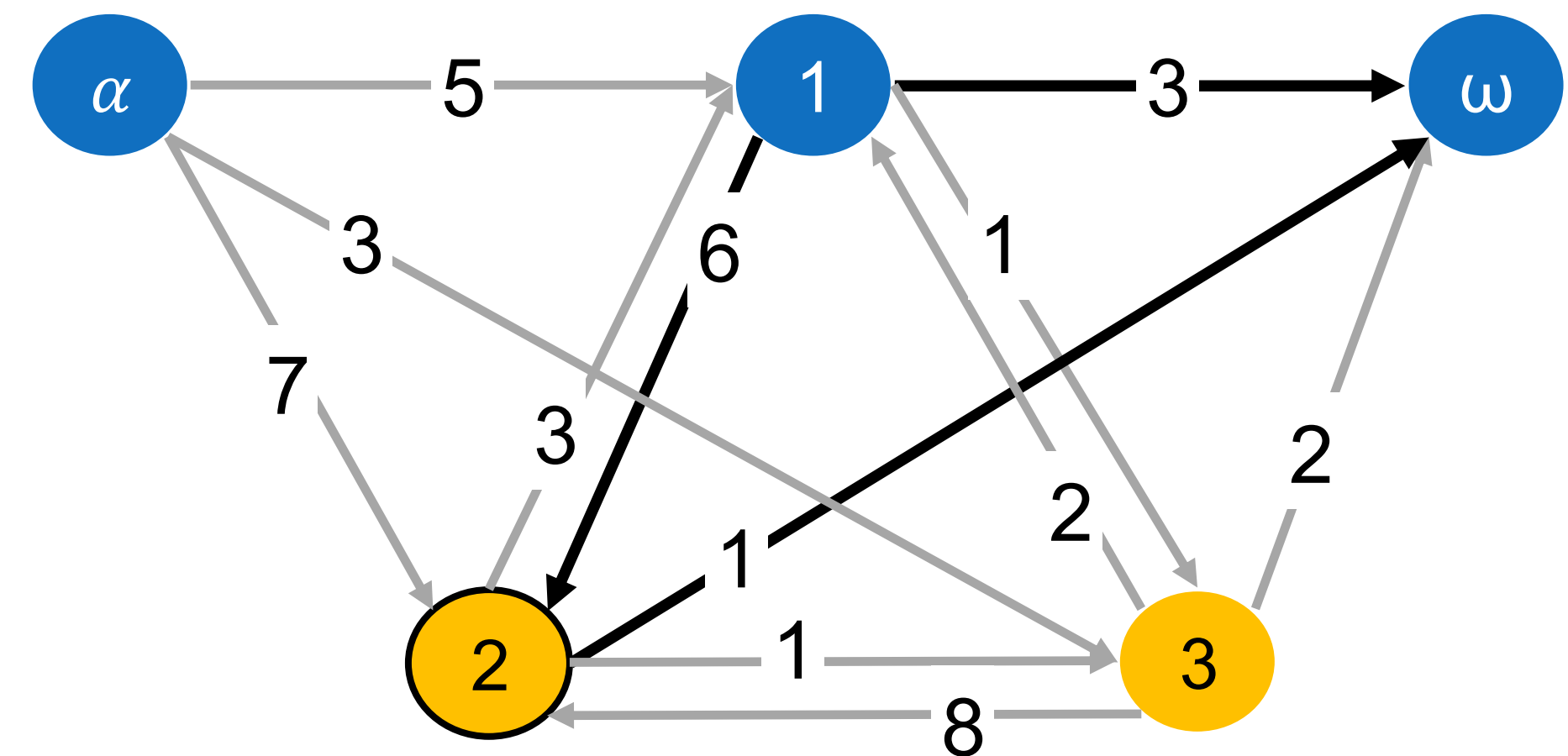
$$\Delta(1, 2, 3) = 6$$



Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\begin{aligned}\Delta(\alpha, 2, 1) &= 5 \\ \Delta(\alpha, 2, 3) &= 5 \\ \Delta(3, 2, 1) &= 9 \\ \Delta(1, 2, 3) &= 6 \\ \Delta(1, 2, \omega) &= 4\end{aligned}$$



Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\Delta(\alpha, 2, 1) = 5$$

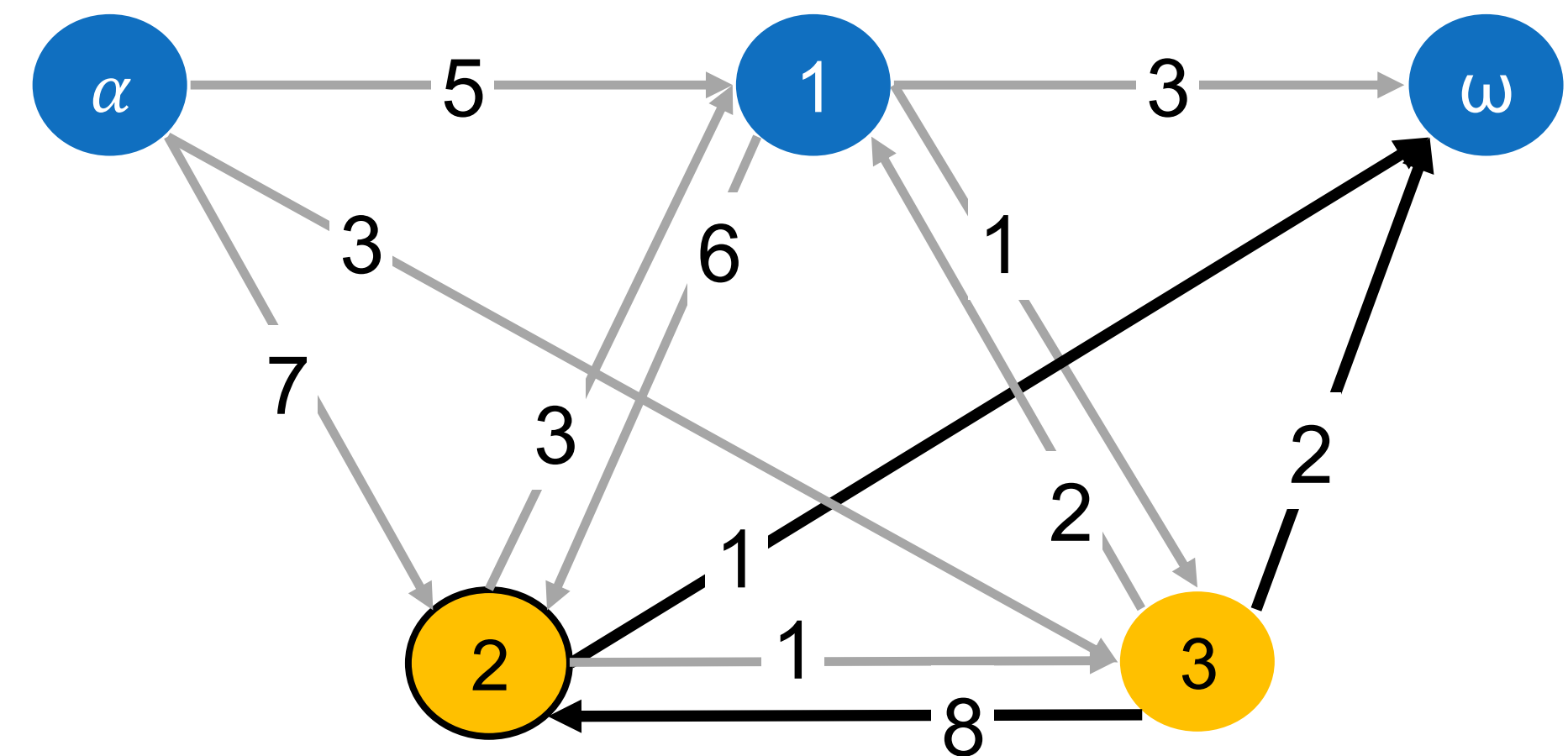
$$\Delta(\alpha, 2, 3) = 5$$

$$\Delta(3, 2, 1) = 9$$

$$\Delta(1, 2, 3) = 6$$

$$\Delta(1, 2, \omega) = 4$$

$$\Delta(3, 2, \omega) = 7$$

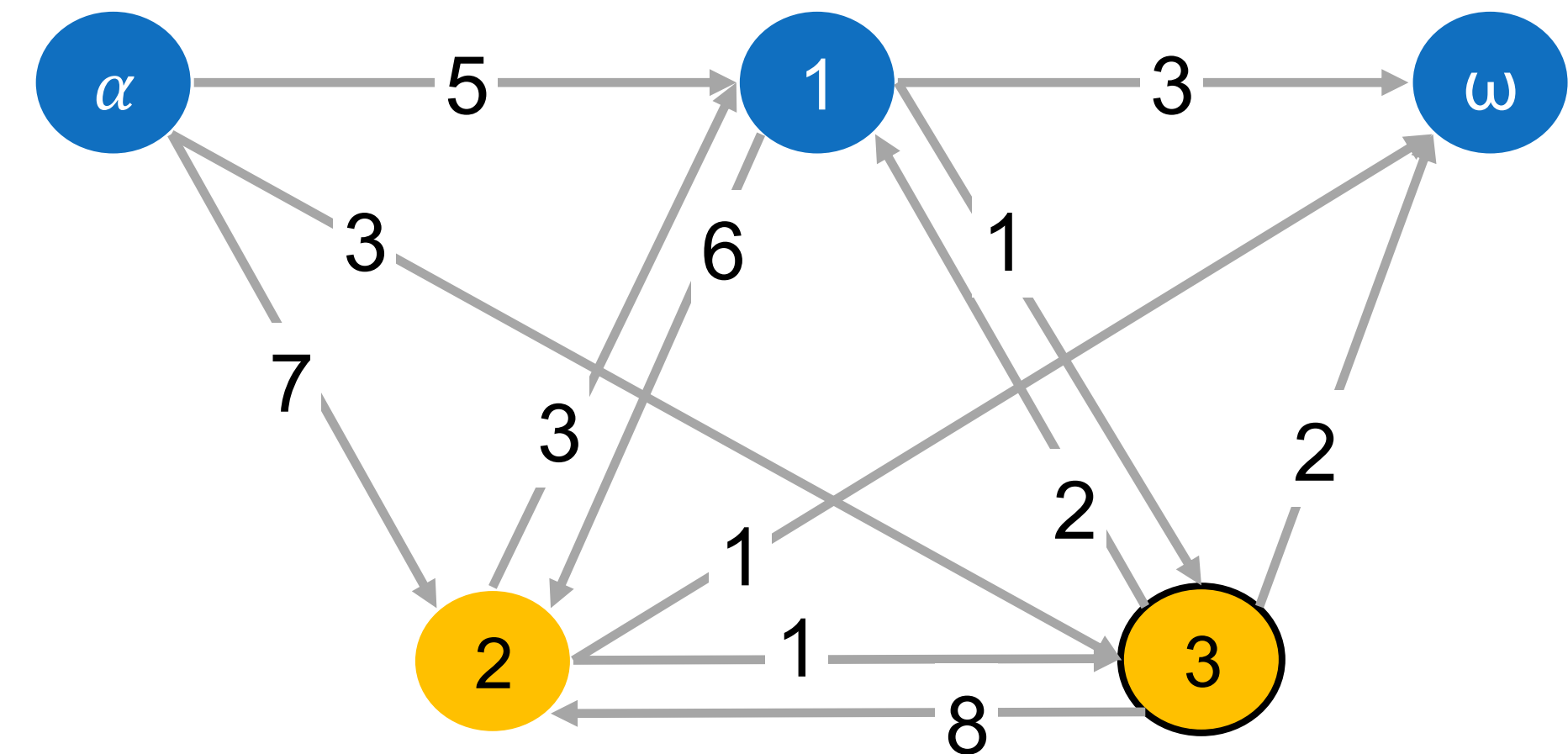


Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) = 8$$

$$\begin{aligned}\Delta(\alpha, 2, 1) &= 5 \\ \Delta(\alpha, 2, 3) &= 5 \\ \Delta(3, 2, 1) &= 9 \\ \Delta(1, 2, 3) &= 6 \\ \Delta(1, 2, \omega) &= 4 \\ \Delta(3, 2, \omega) &= 7\end{aligned}$$

$$\begin{aligned}\Delta(\alpha, 3, 1) &= 0 \\ \Delta(\alpha, 3, 2) &= 4 \\ \Delta(1, 3, 2) &= 3 \\ \Delta(2, 3, 1) &= 0 \\ \Delta(1, 3, \omega) &= 0 \\ \Delta(2, 3, \omega) &= 2\end{aligned}$$

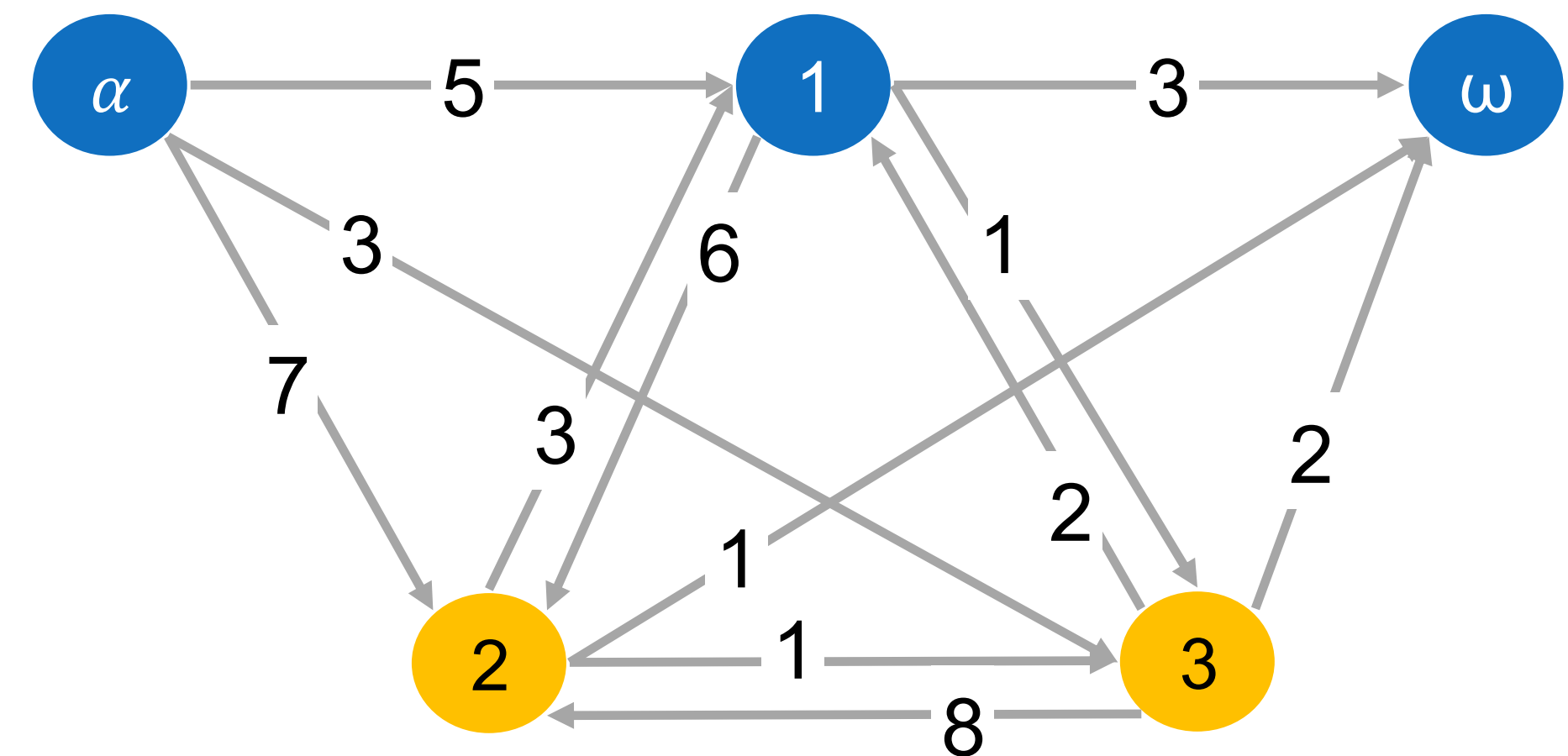


Lower Bound Based on Detours

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

$$\begin{aligned}\Delta(\alpha, 2, 1) &= 5 \\ \Delta(\alpha, 2, 3) &= 5 \\ \Delta(3, 2, 1) &= 9 \\ \Delta(1, 2, 3) &= 6 \\ \Delta(1, 2, \omega) &= 4 \\ \Delta(3, 2, \omega) &= 7\end{aligned}$$

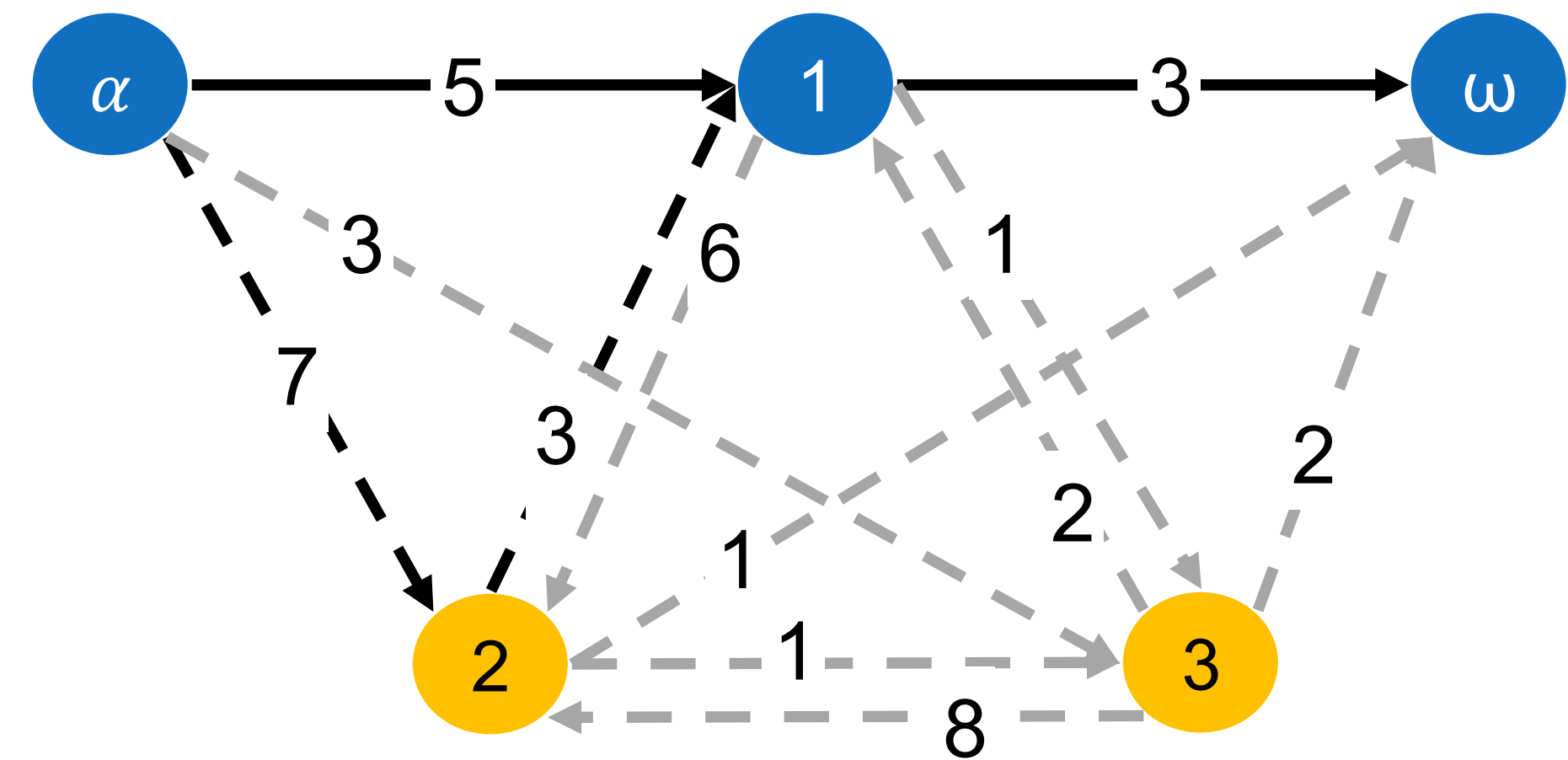
$$\begin{aligned}\Delta(\alpha, 3, 1) &= 0 \\ \Delta(\alpha, 3, 2) &= 4 \\ \Delta(1, 3, 2) &= 3 \\ \Delta(2, 3, 1) &= 0 \\ \Delta(1, 3, \omega) &= 0 \\ \Delta(2, 3, \omega) &= 2\end{aligned}$$



Lower Bound Based on Detours

Filtering

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

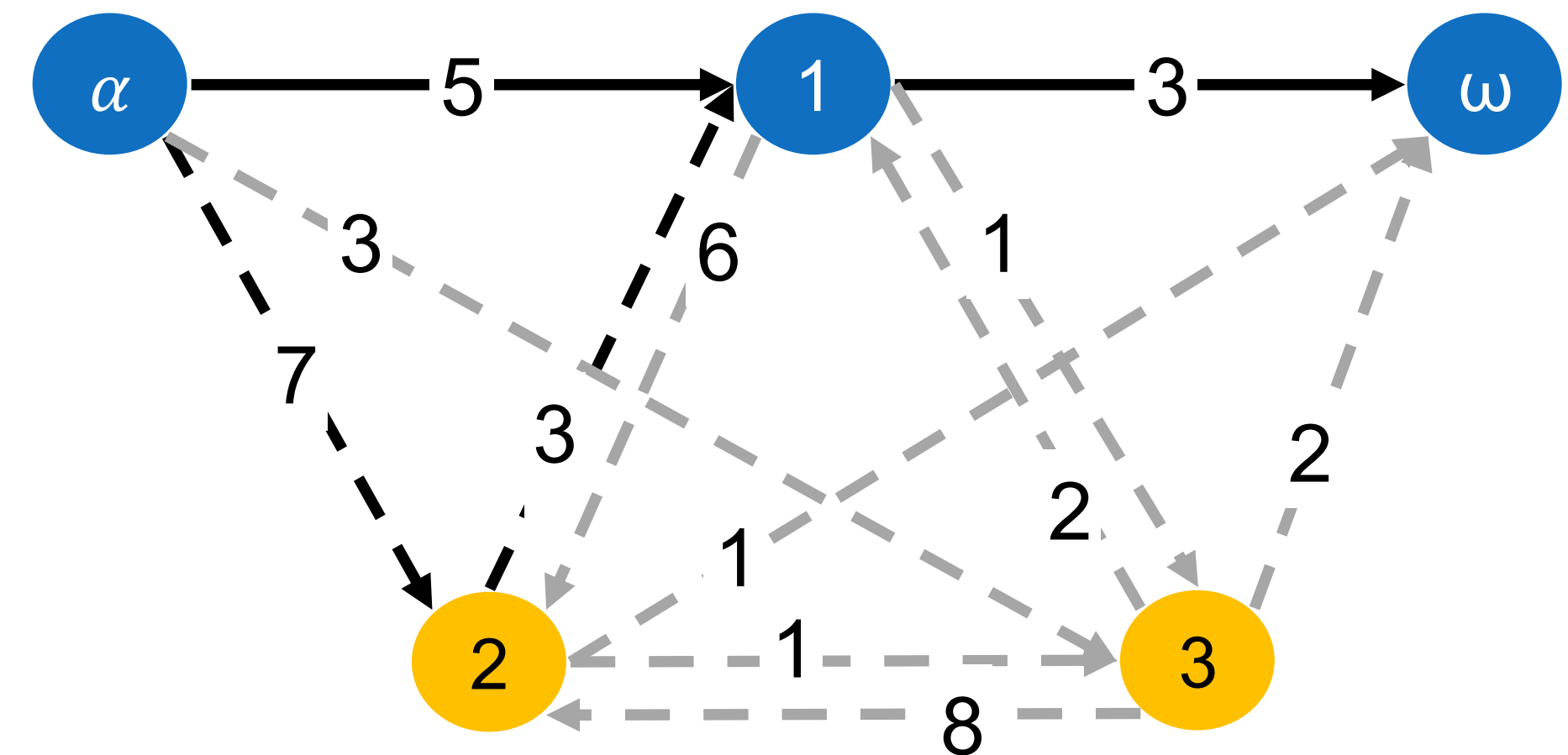


Lower Bound Based on Detours

Filtering

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

$$\text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(\alpha, 3, 1)$$

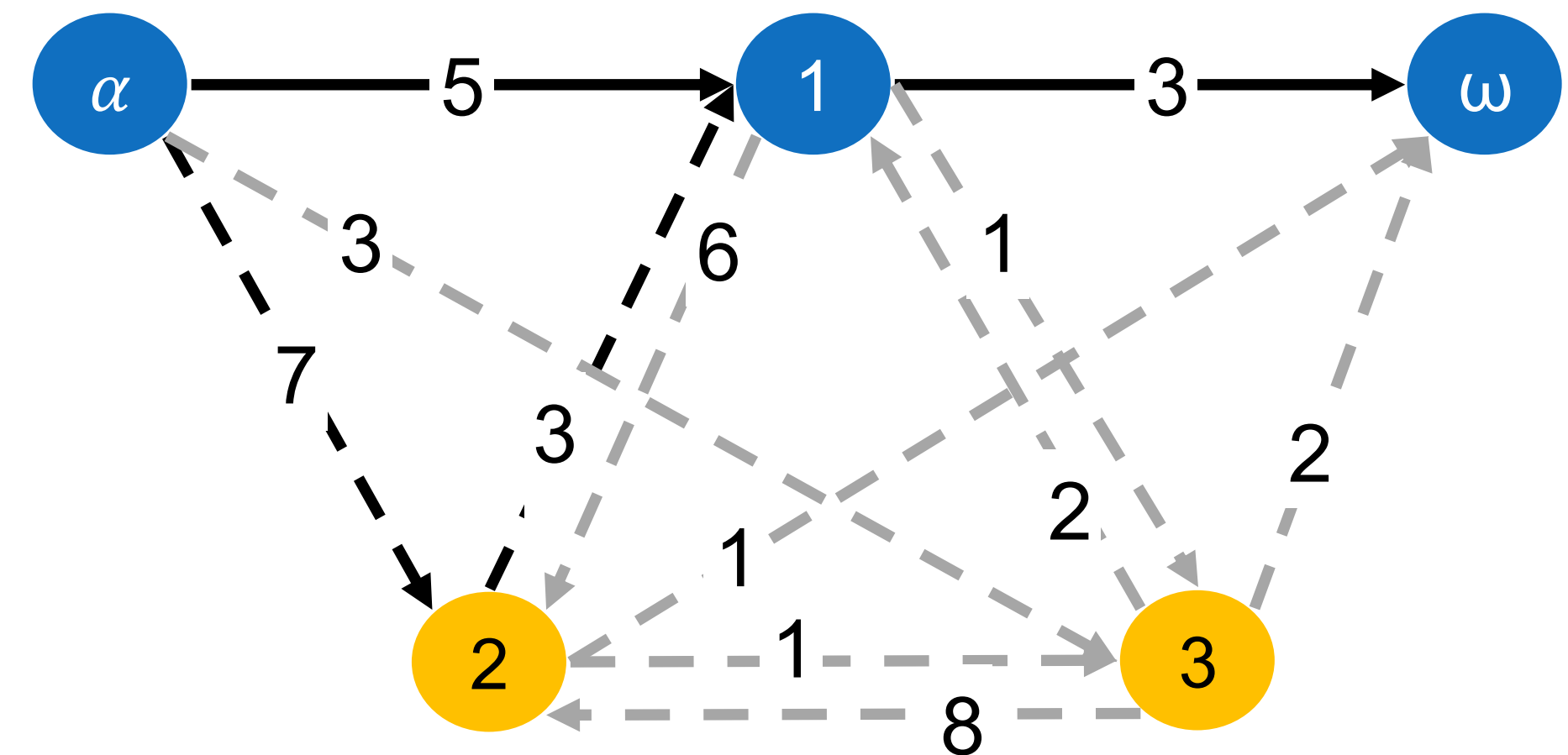


Lower Bound Based on Detours

Filtering

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

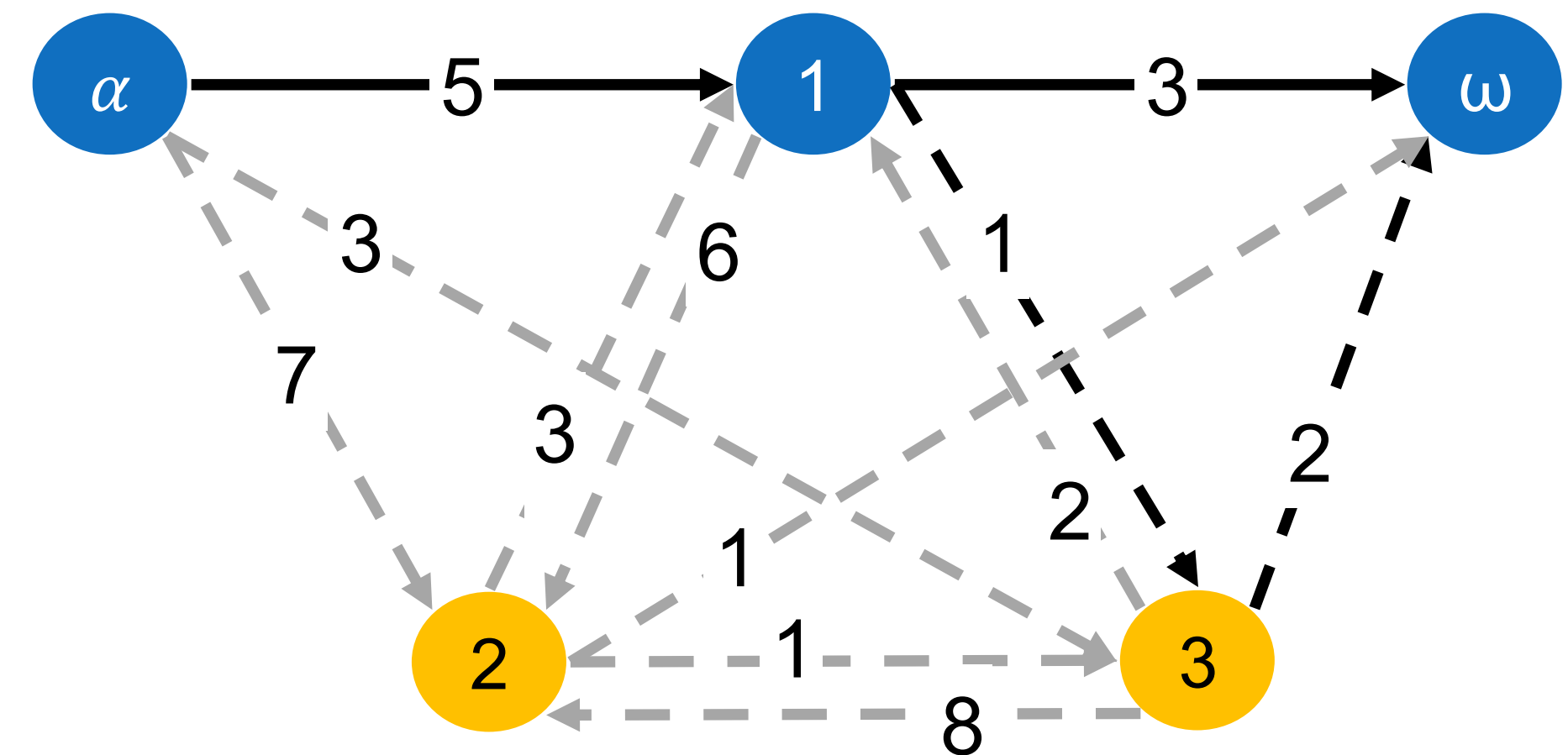
$$\text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(\alpha, 3, 1) + \Delta(\alpha, 2, 1) = 13$$



Lower Bound Based on Detours

Filtering

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

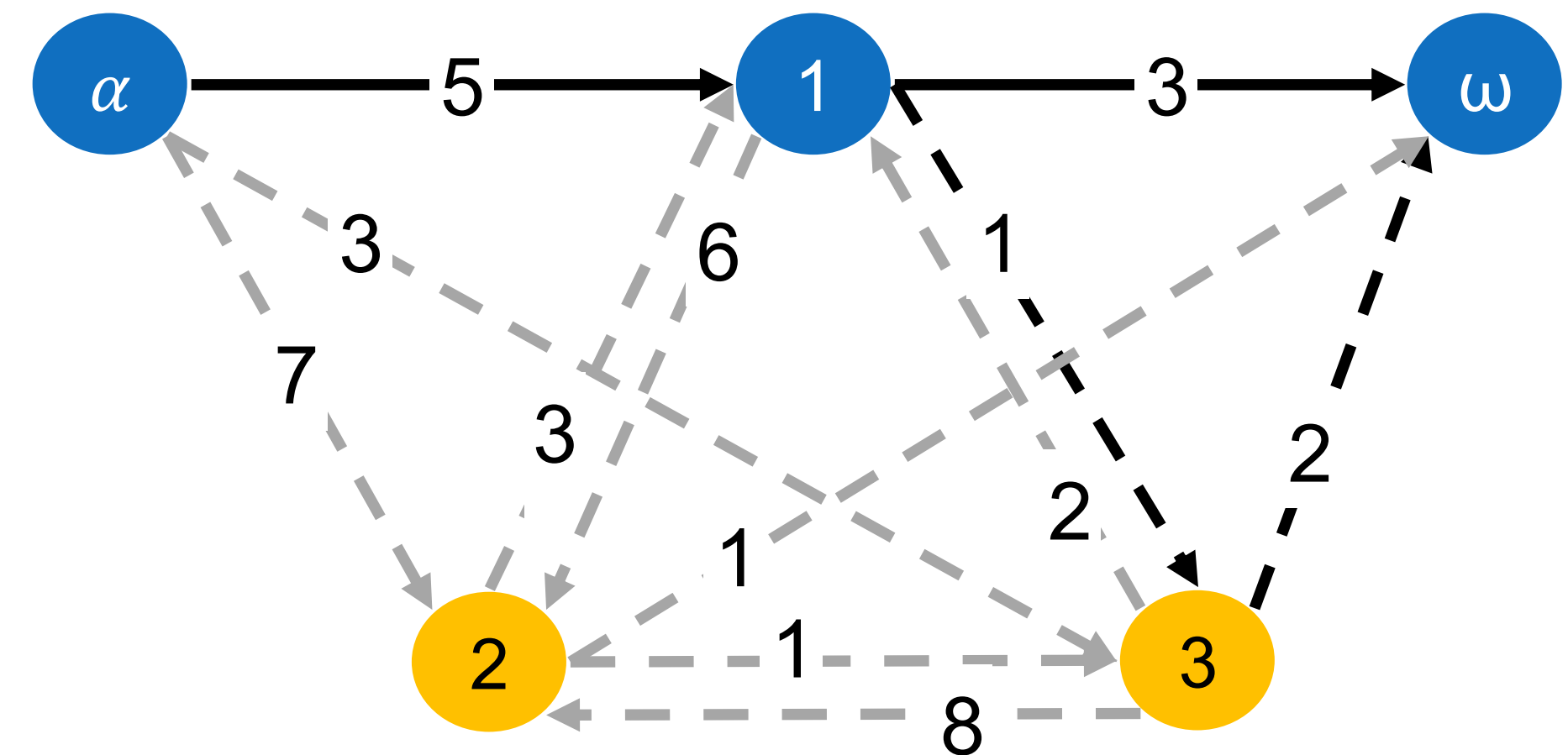


Lower Bound Based on Detours

Filtering

$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

$$\text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega)$$

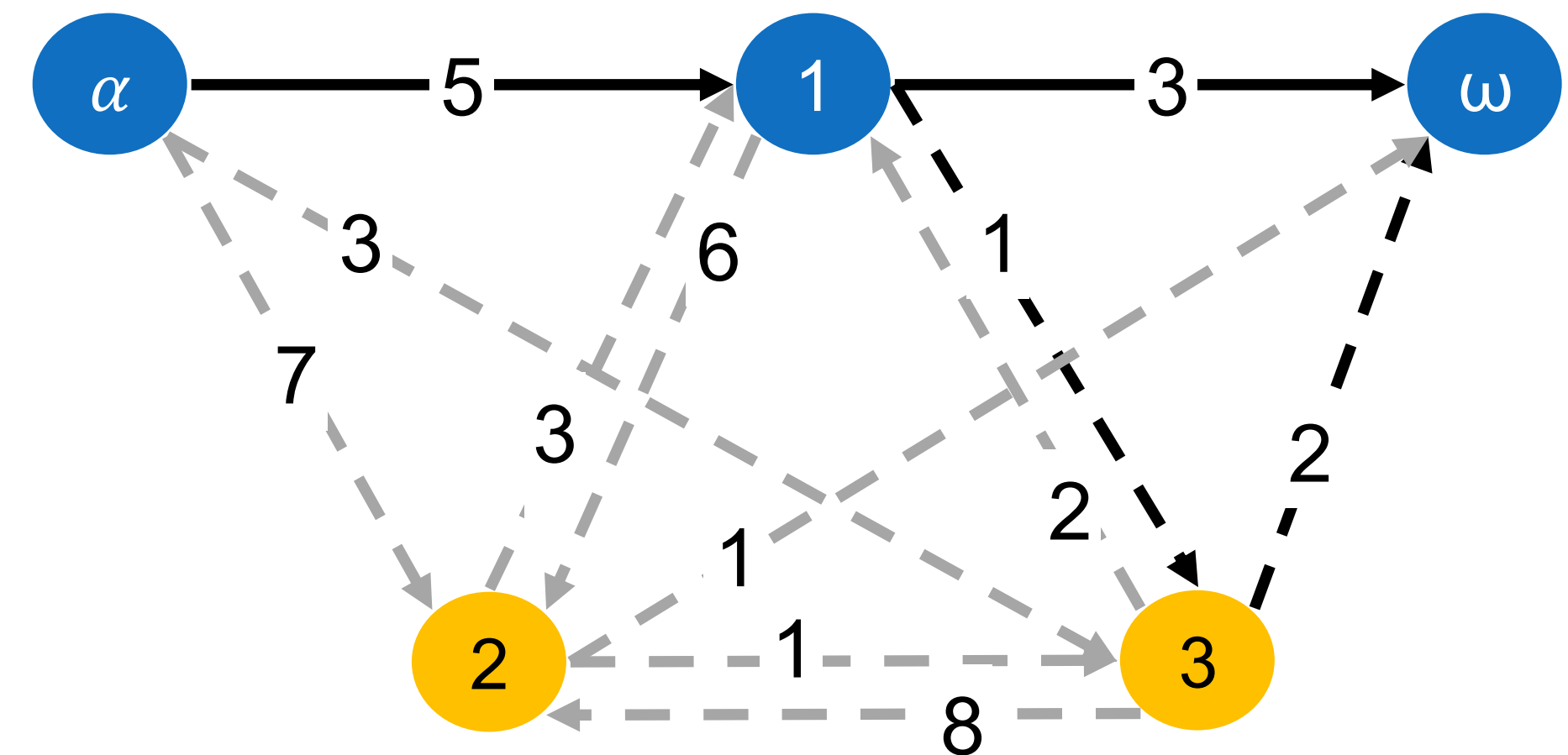


Lower Bound Based on Detours

Filtering

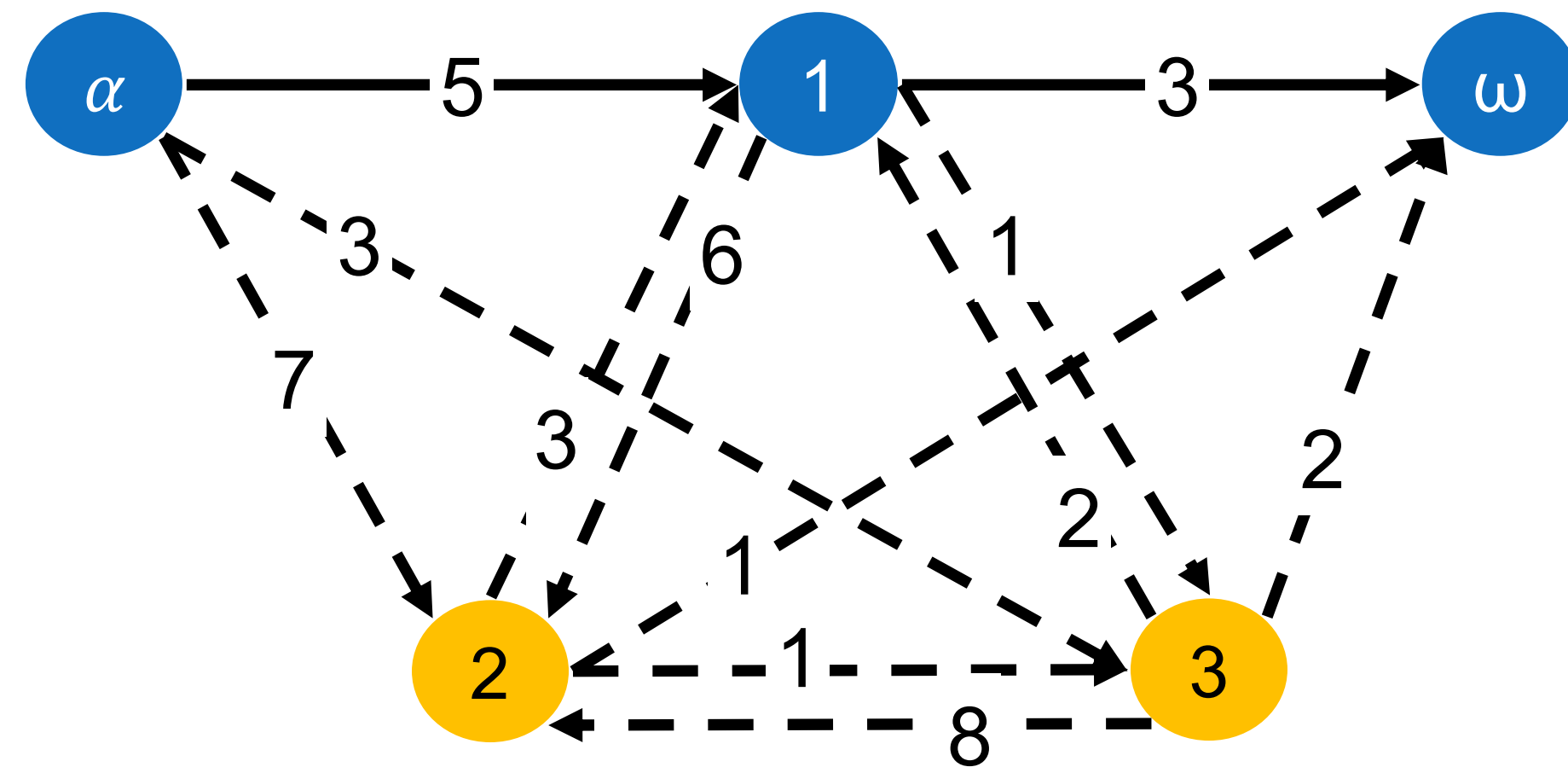
$$L_D = \text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(\alpha, 3, 1) = 12$$

$$\text{cost}(\alpha, 1) + \text{cost}(1, \omega) + \Delta(1, 2, \omega) + \Delta(1, 3, \omega) = 12$$



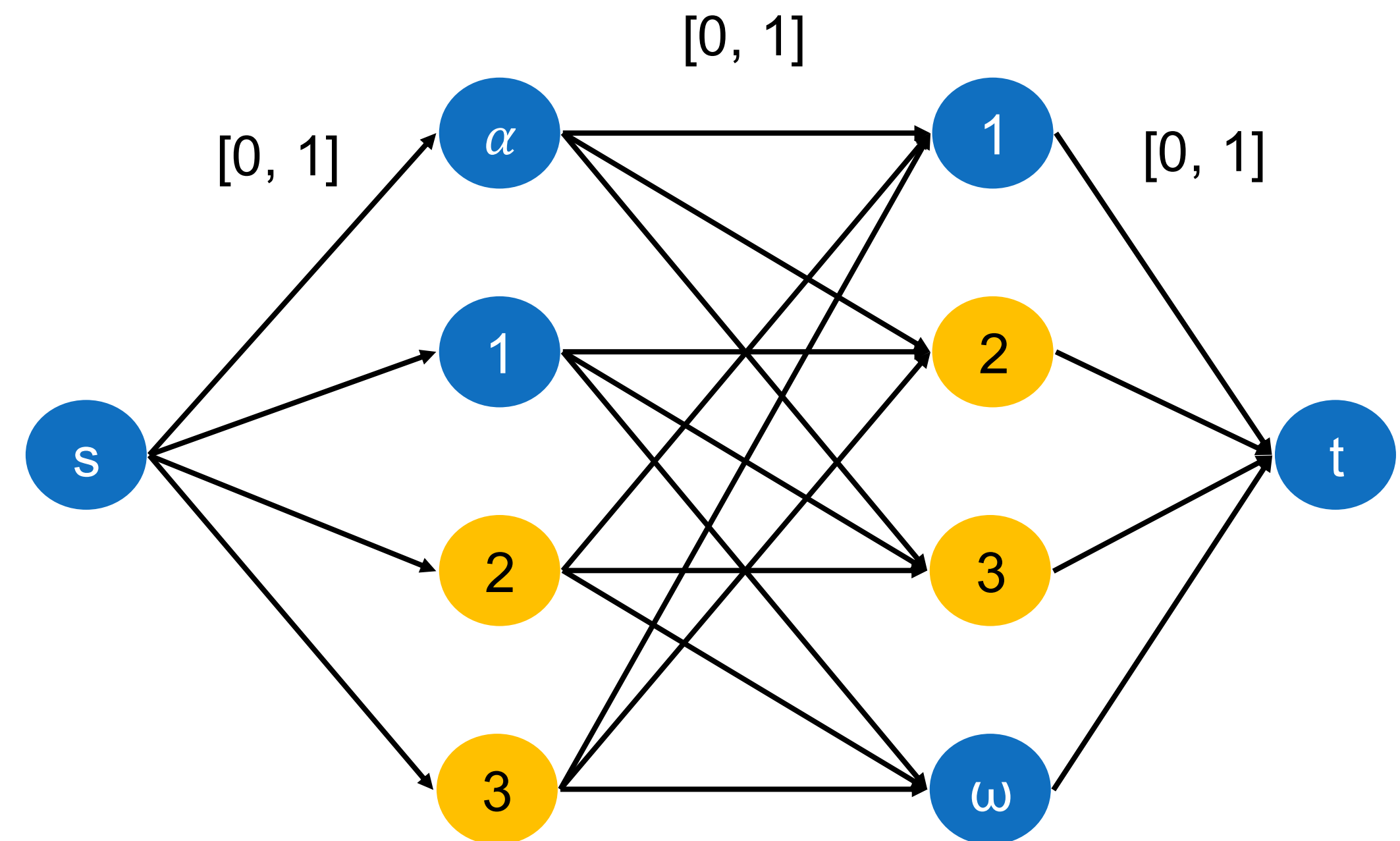
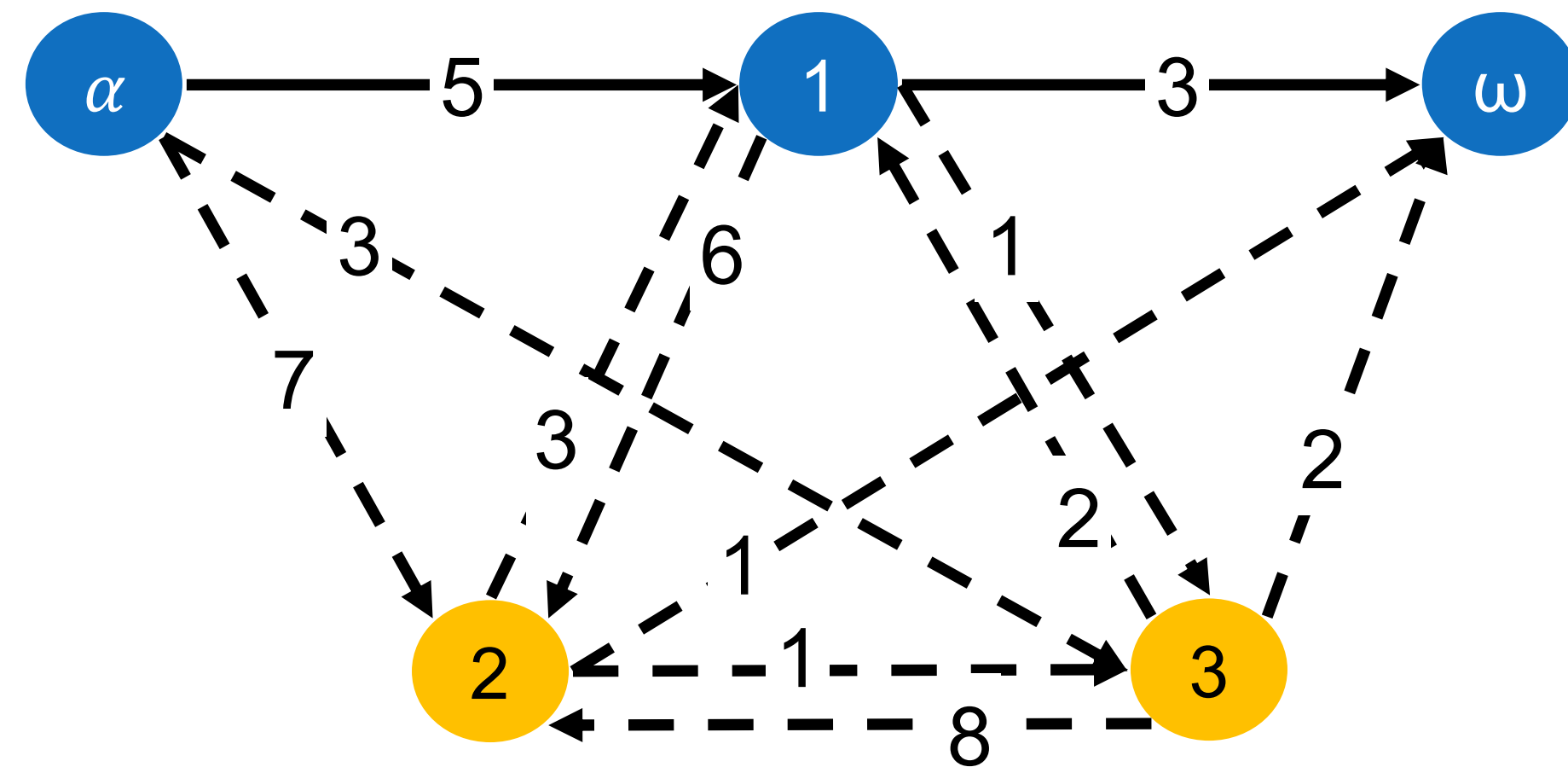
Lower bound based on matching

Lower bound based on matching

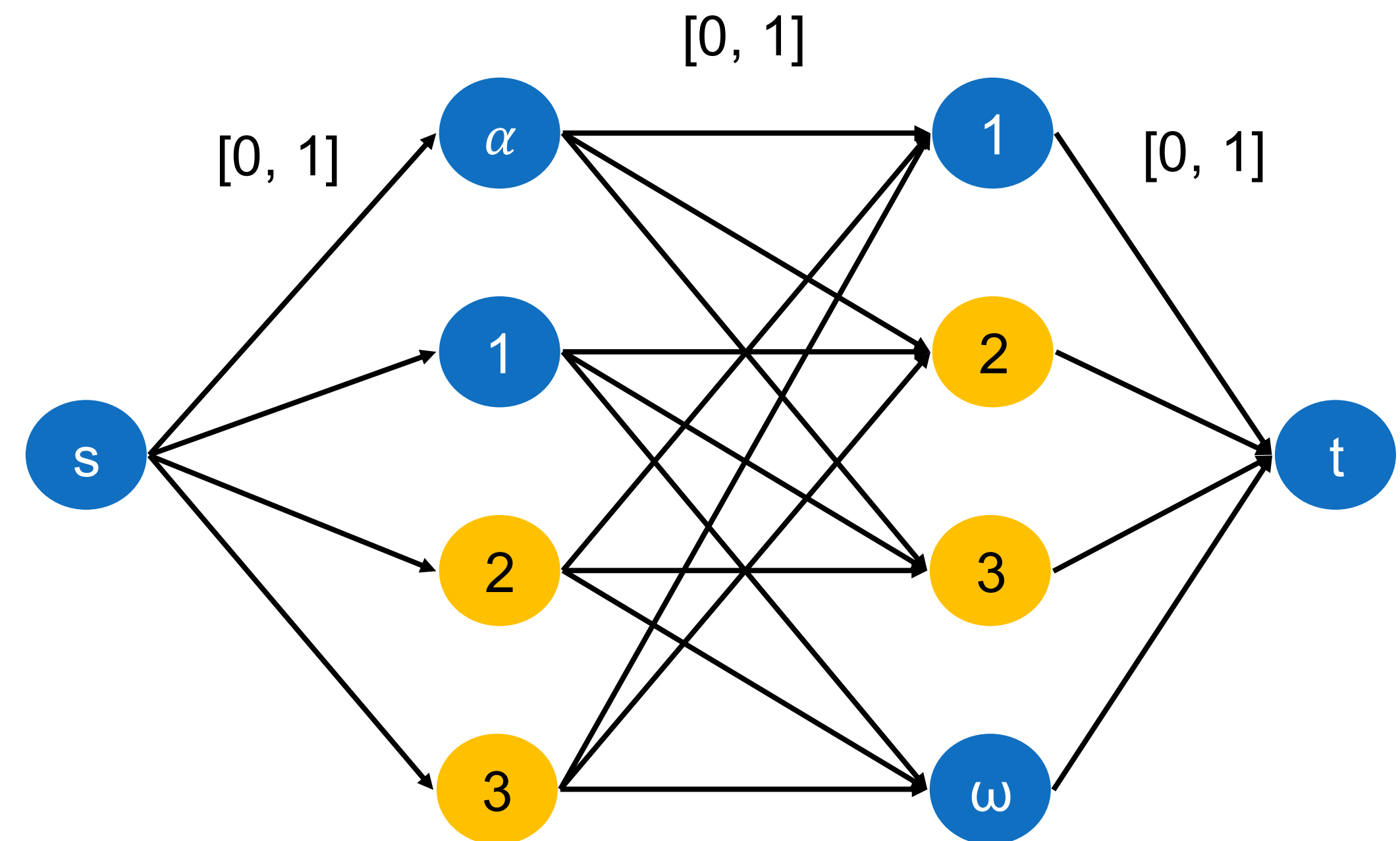
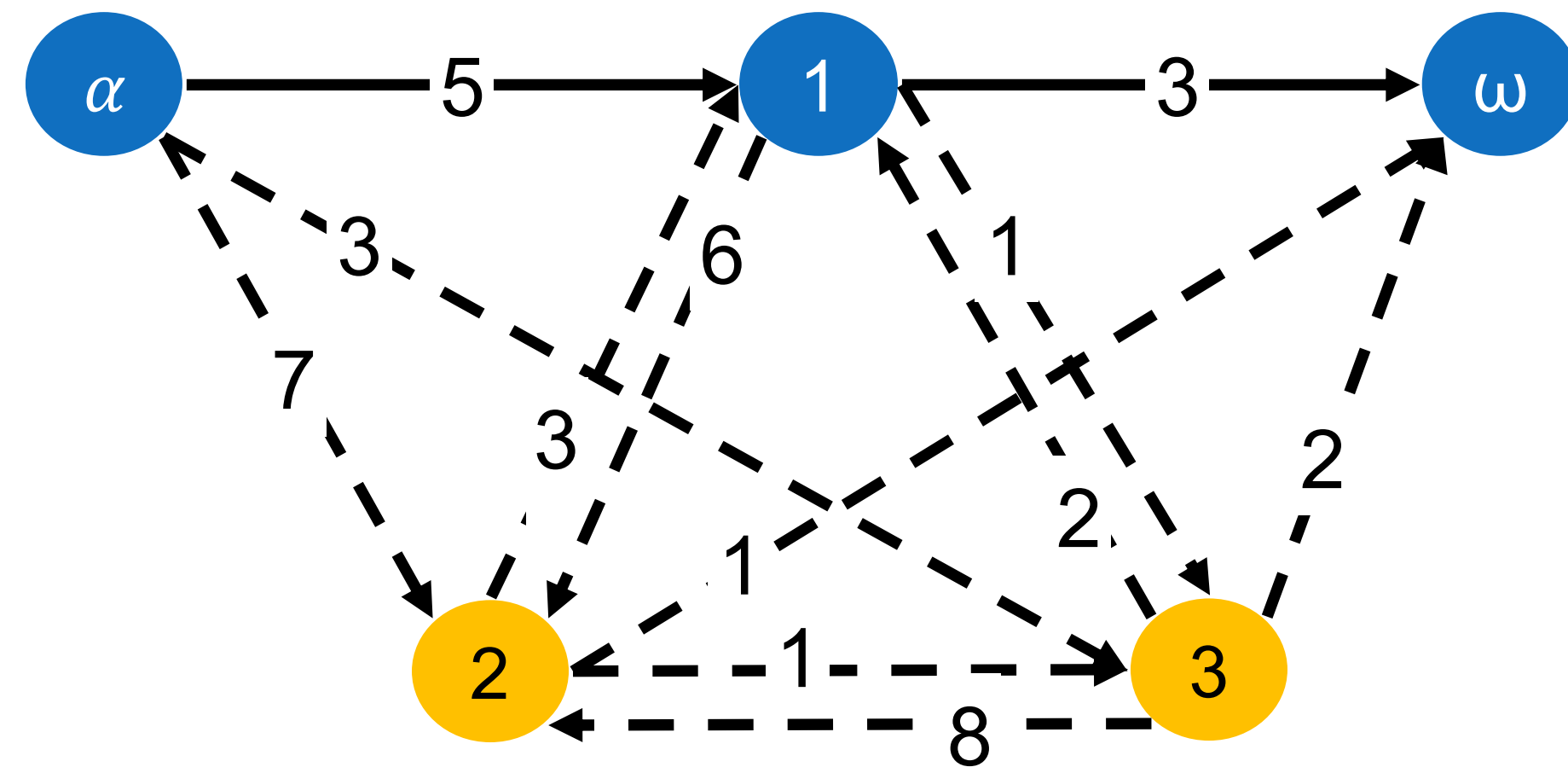


The Distance Constraint on Sequence Variables

Lower bound based on matching

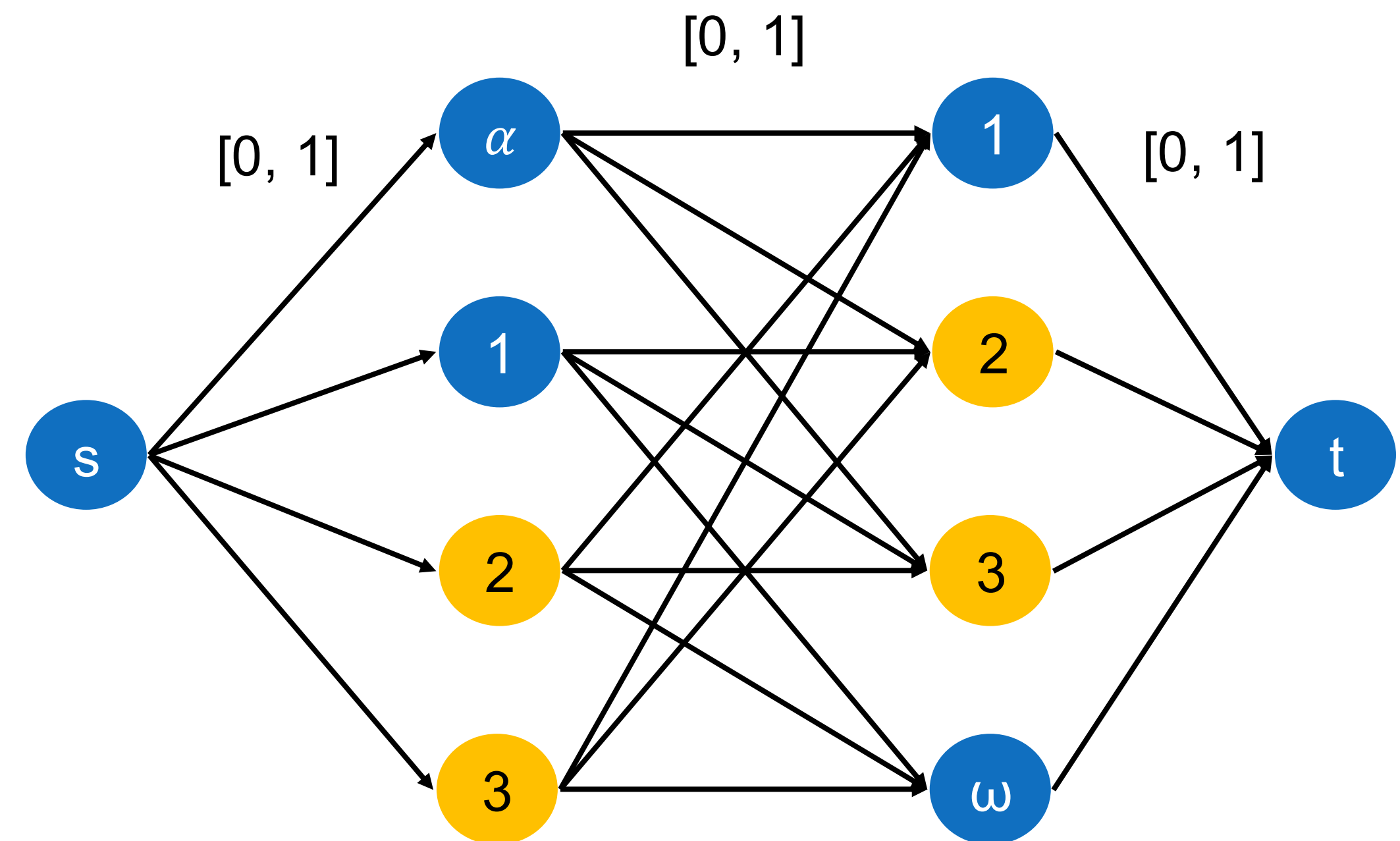
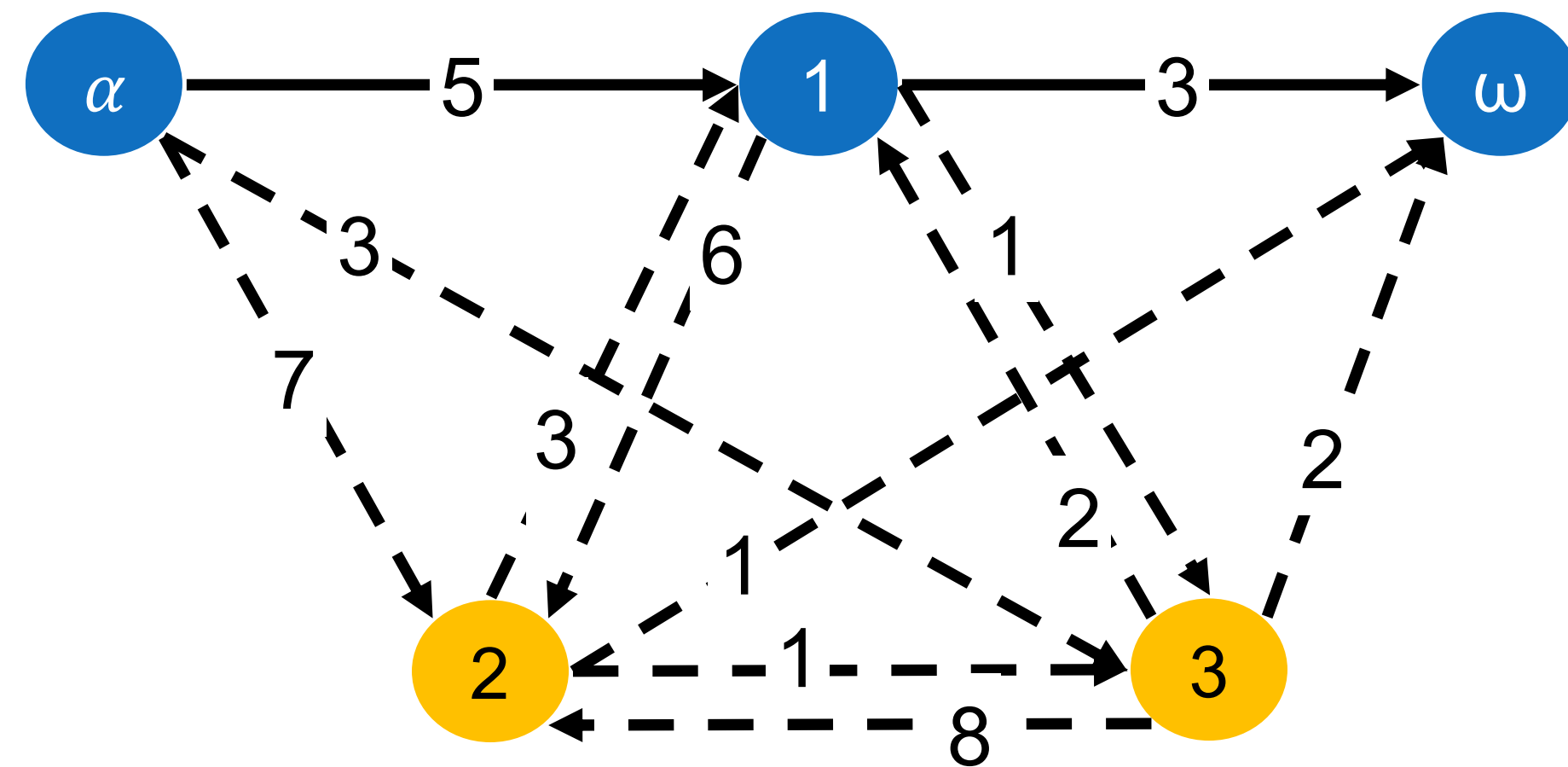


Lower bound based on matching



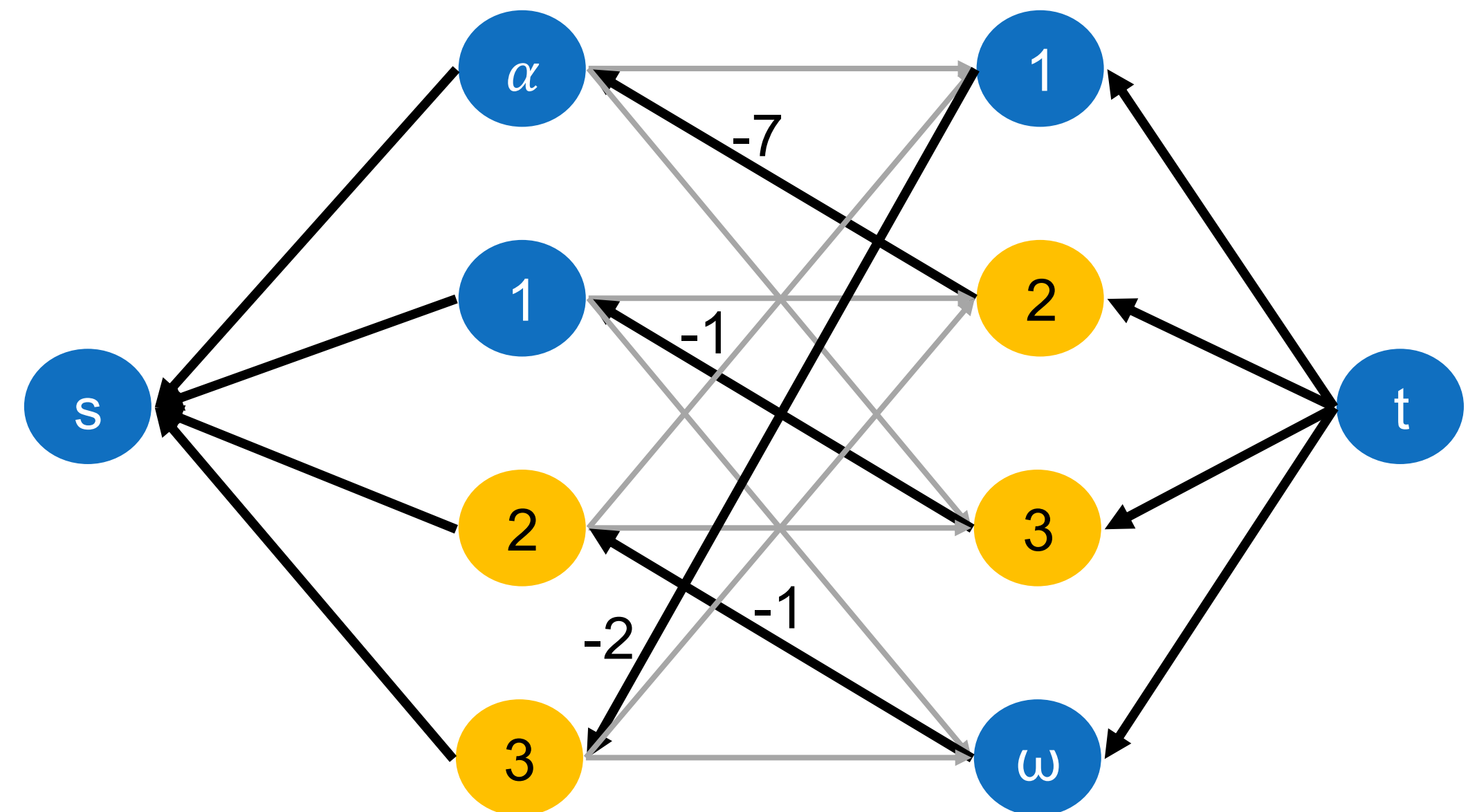
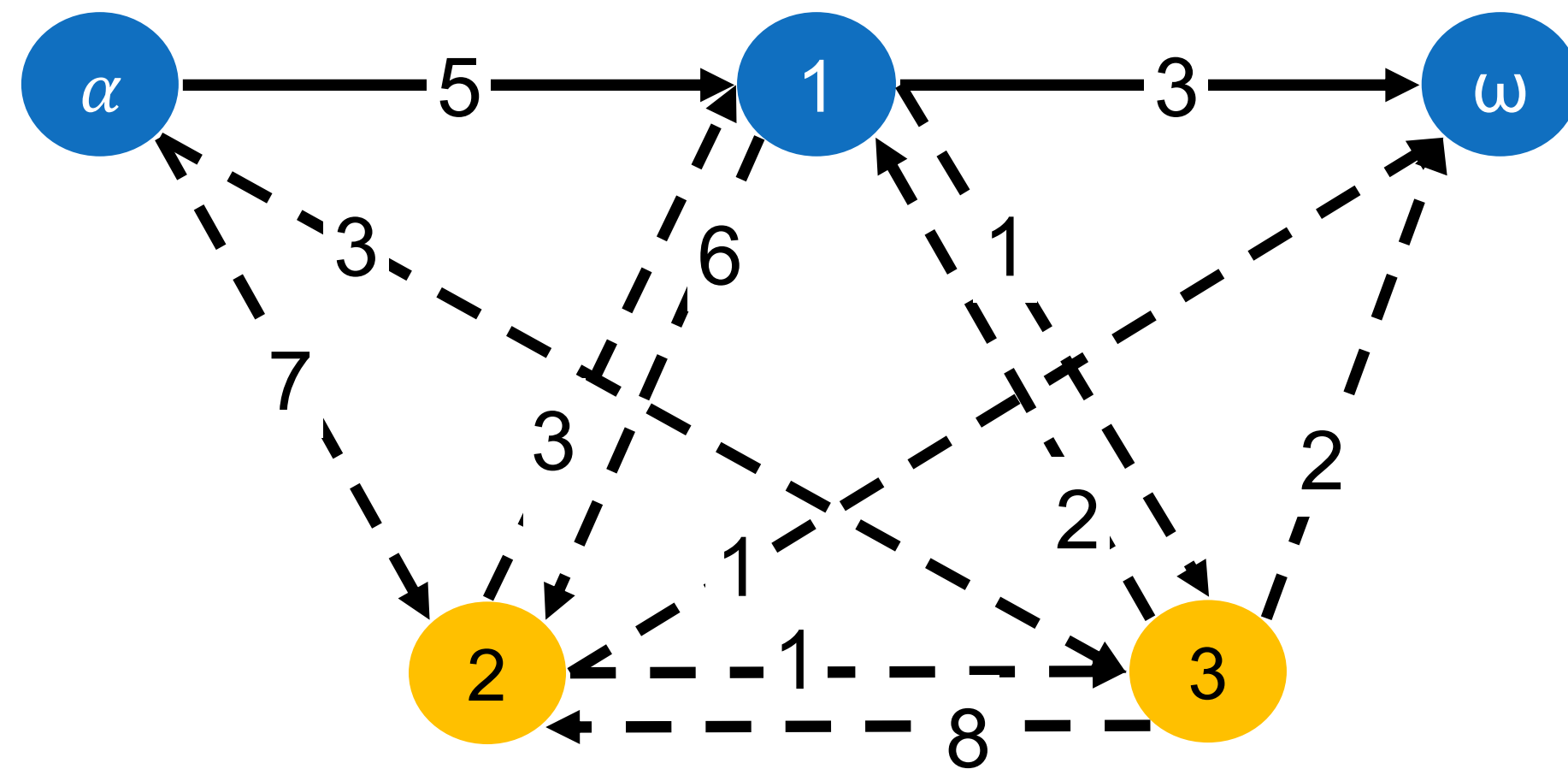
We add the transition costs to the arcs

Lower bound based on matching



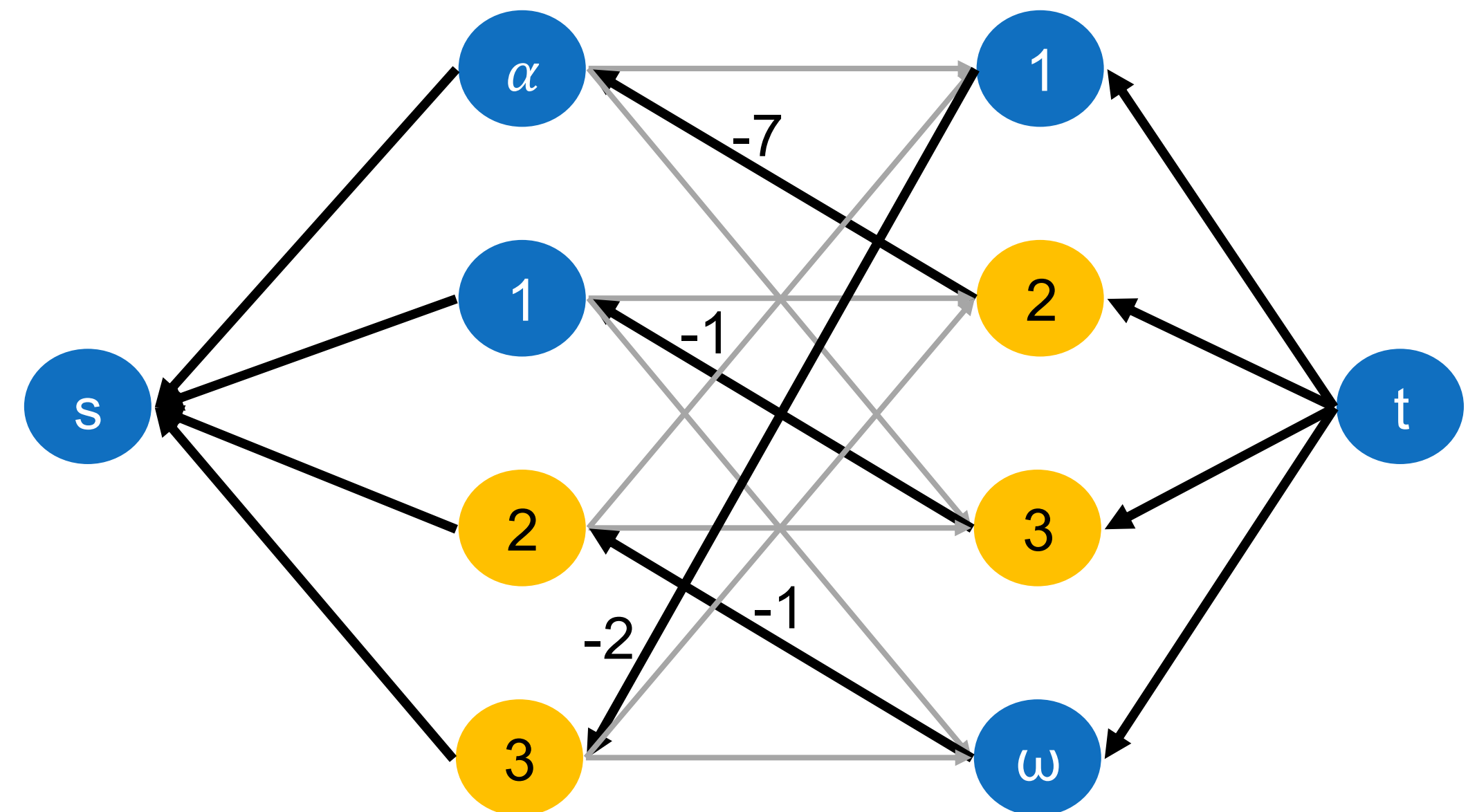
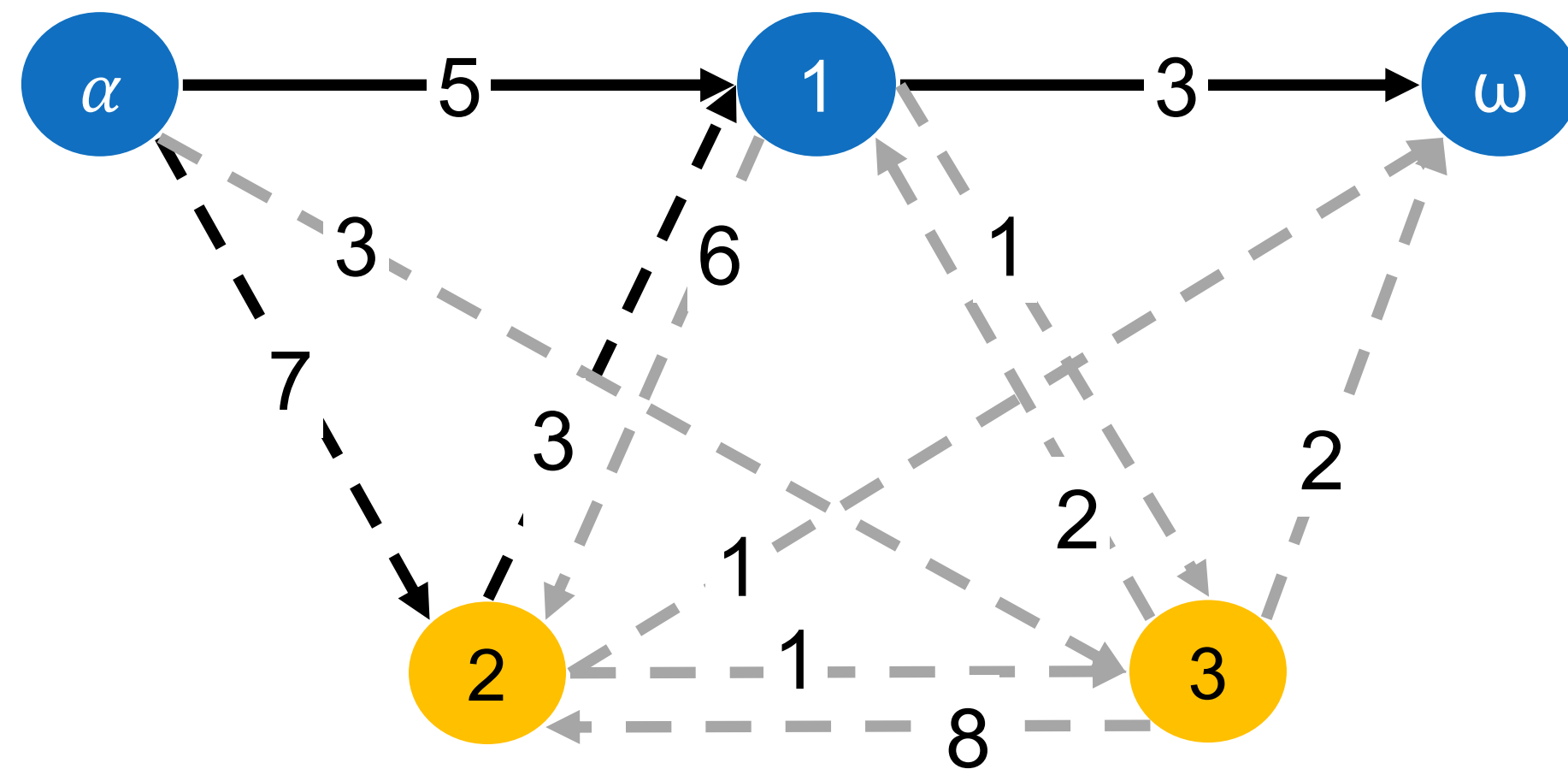
We apply the MinCostMaxFlow algorithm

Lower bound based on matching



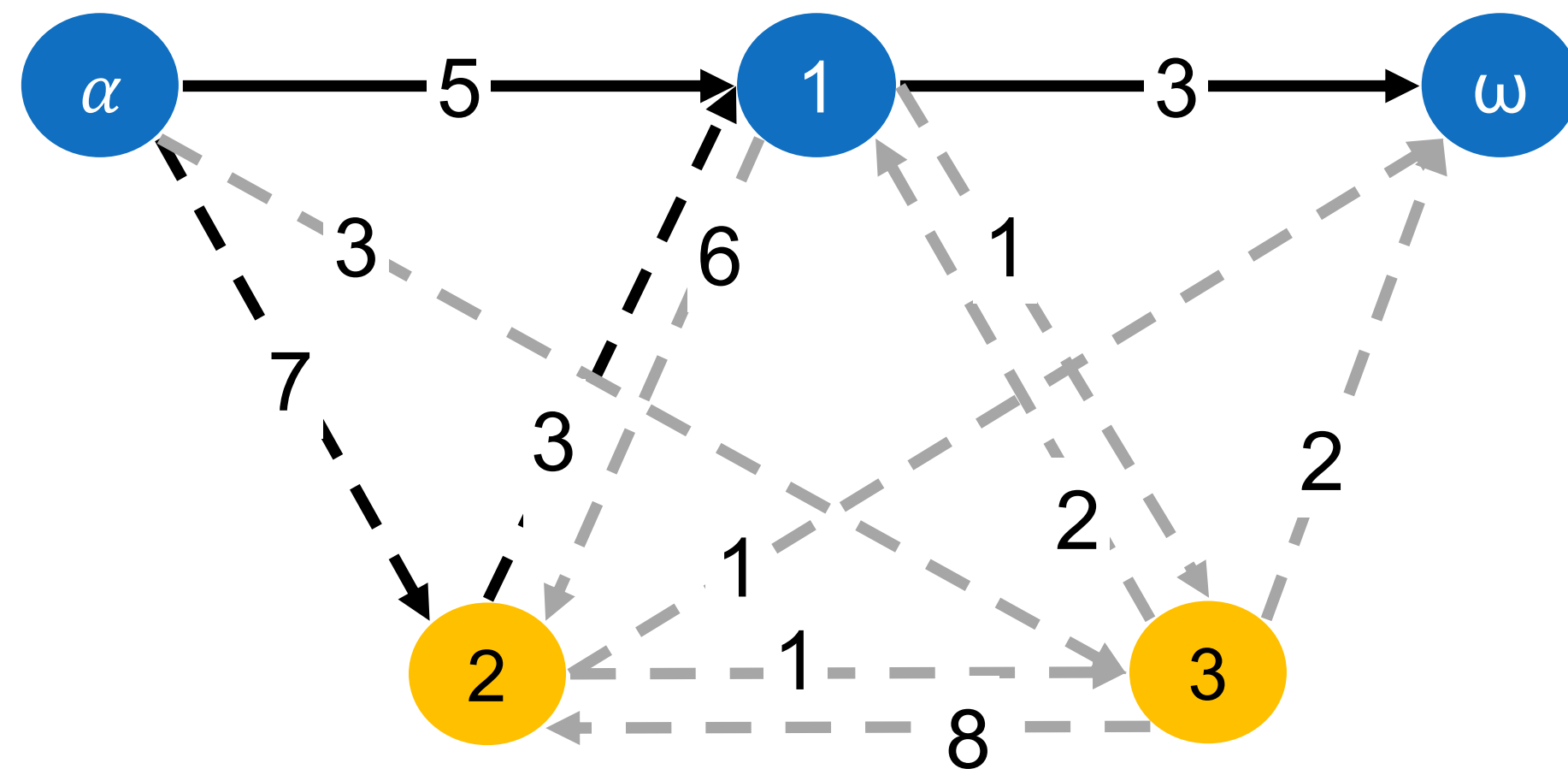
$$\begin{aligned} L_{MS} &= \text{cost}(\text{flow}) \\ &= 7 + 1 + 1 + 2 = 11 \end{aligned}$$

Lower bound based on matching



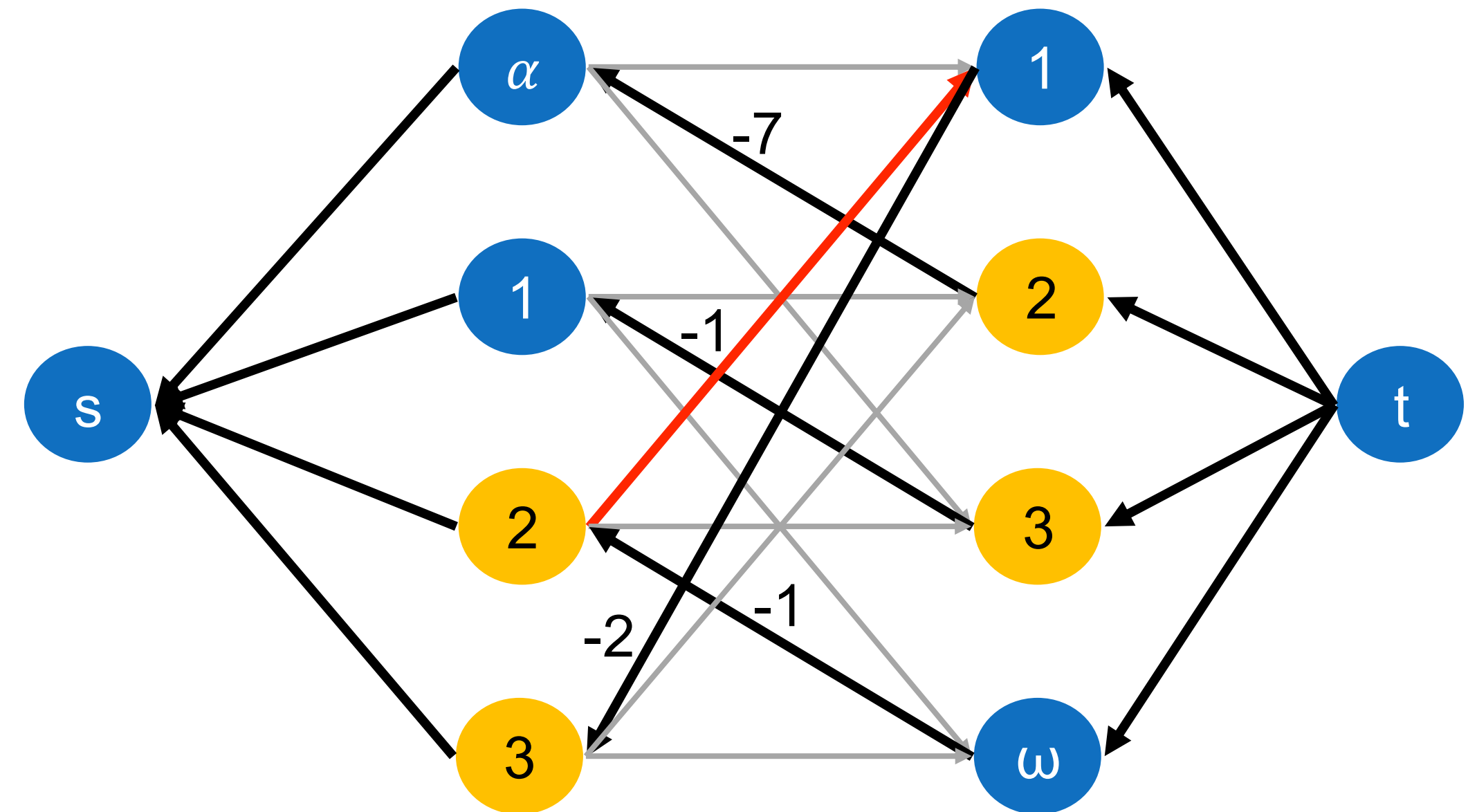
$$\begin{aligned} L_{MS} &= \text{cost}(\text{flow}) \\ &= 7 + 1 + 1 + 2 = 11 \end{aligned}$$

Lower bound based on matching



$$L_{MS} = \text{cost}(\text{flow})$$

$$= 7 + 1 + 1 + 2 = 11$$



$$SP(1, 2) > \text{Dist} - L_{MD} - \text{cost}(2, 1)$$

$$-1 > 12 - 11 - 3$$

$$-1 > -2$$

Results

Results

	L_O	L_{MIO}	L_{MD}	L_{MS}
L_O		×	✓	×
L_{MIO}	×		×	✓
L_{MD}	×	×		×
L_{MS}	×	×	×	

The Distance Constraint on Sequence Variables

Results

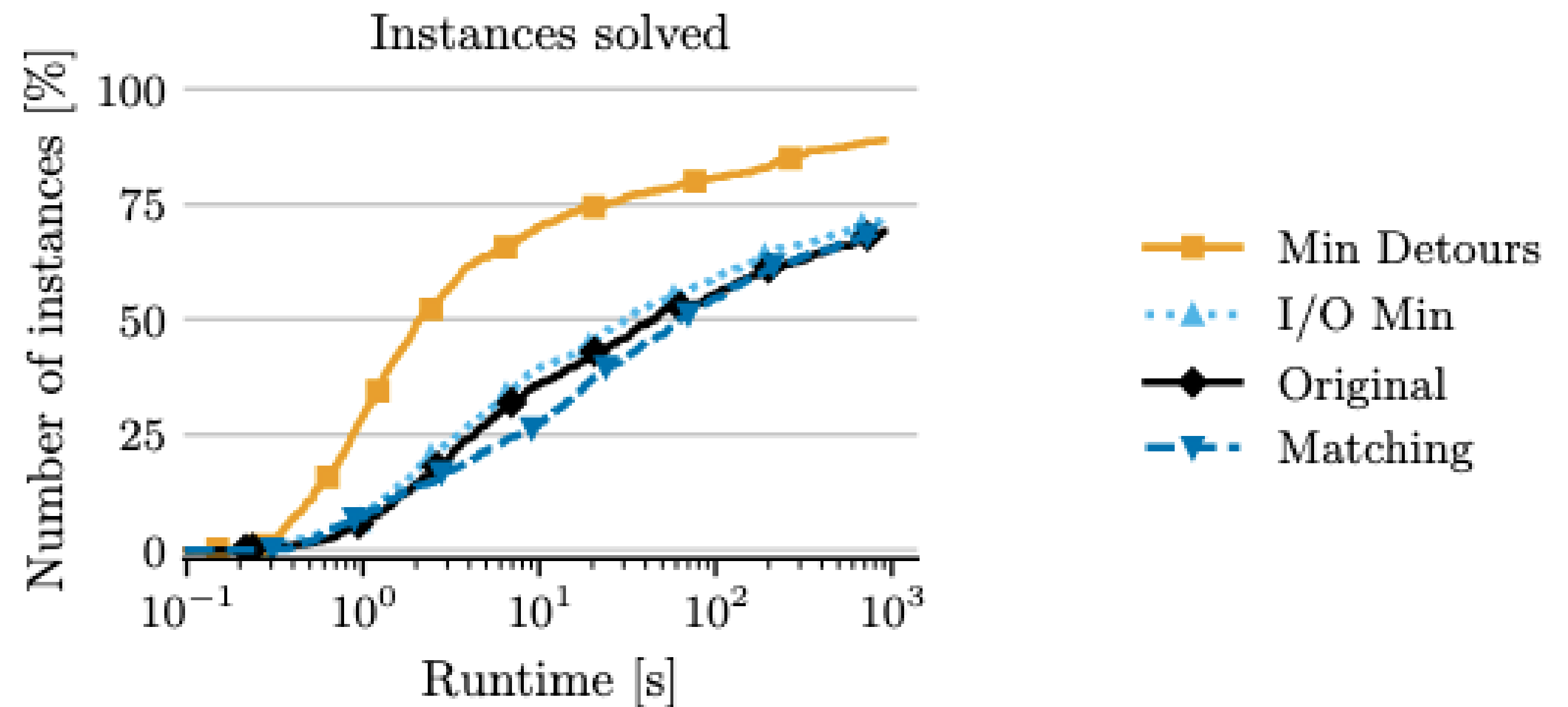
MaxiCP

TourMustSee problem: Maximize collection profit subject to a distance constraint; certain vertices are required and others are optional

1020 instances ranging from 20 to 38 nodes.



The Distance Constraint on Sequence Variables



Results

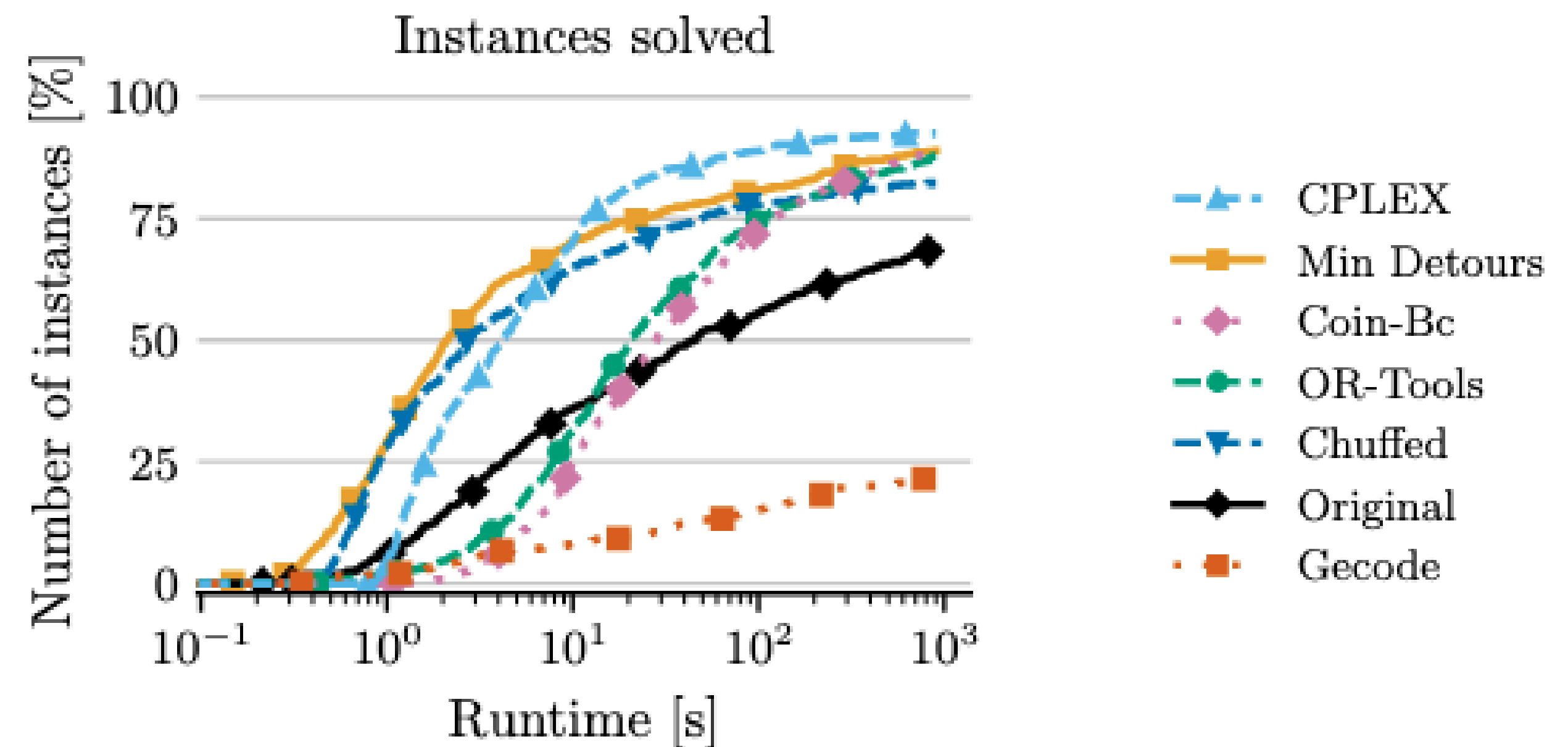
MaxiCP

TourMustSee problem: Maximize collection profit subject to a distance constraint; certain vertices are required and others are optional

1020 instances ranging from 20 to 38 nodes.



The Distance Constraint on Sequence Variables



Conclusion



3 new lower bounds



The bound with the
detours strictly dominate
the original



Performance
Improvement



Public constraints

Thank you for your attention!



GitHub link



Paper link