

# Optimizing a Multi-Commodity Home-Delivery and Pickup Service in Depopulated Rural Areas with Constraint Programming

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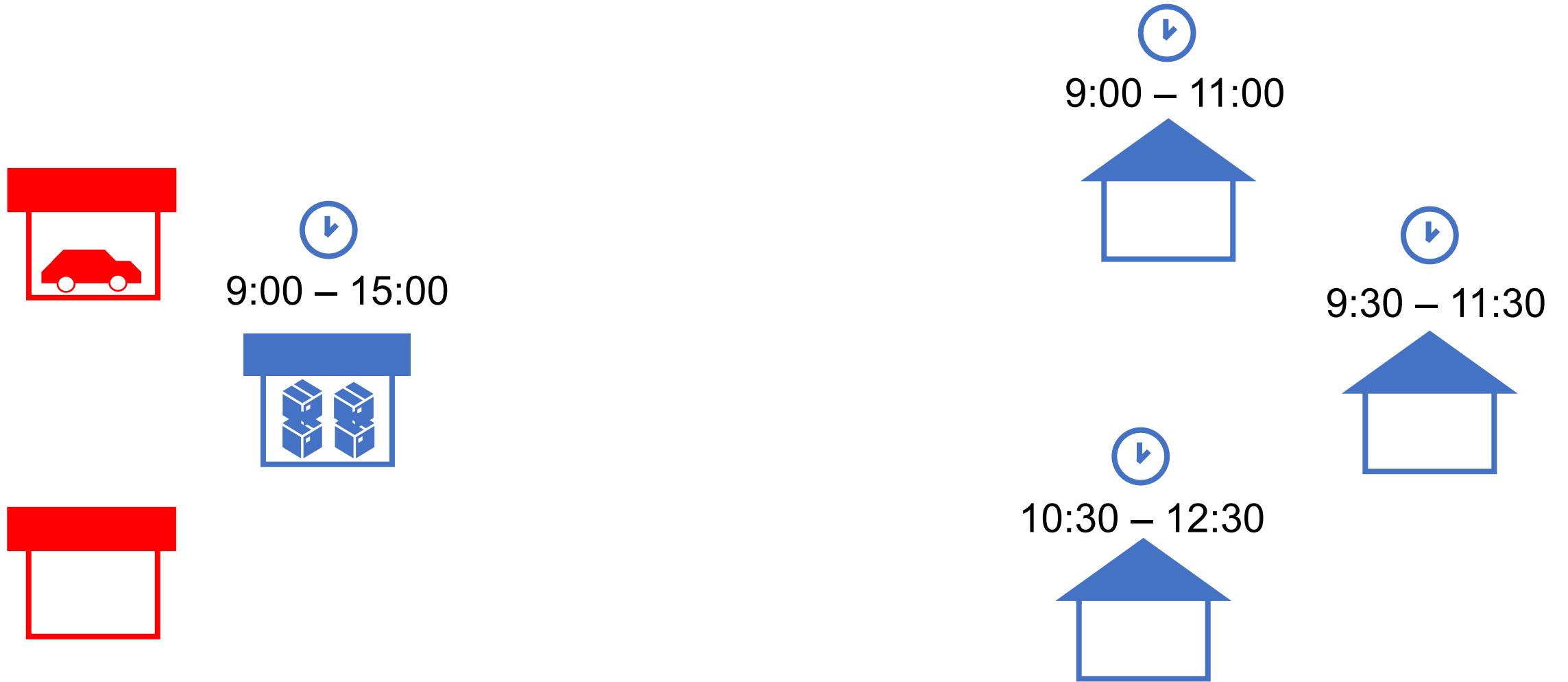
# Transportation in Depopulated Rural Areas

- Poor access to public transportation
- Driving is difficult for elderly people
- Home-delivery and pickup service with a limited number of drivers

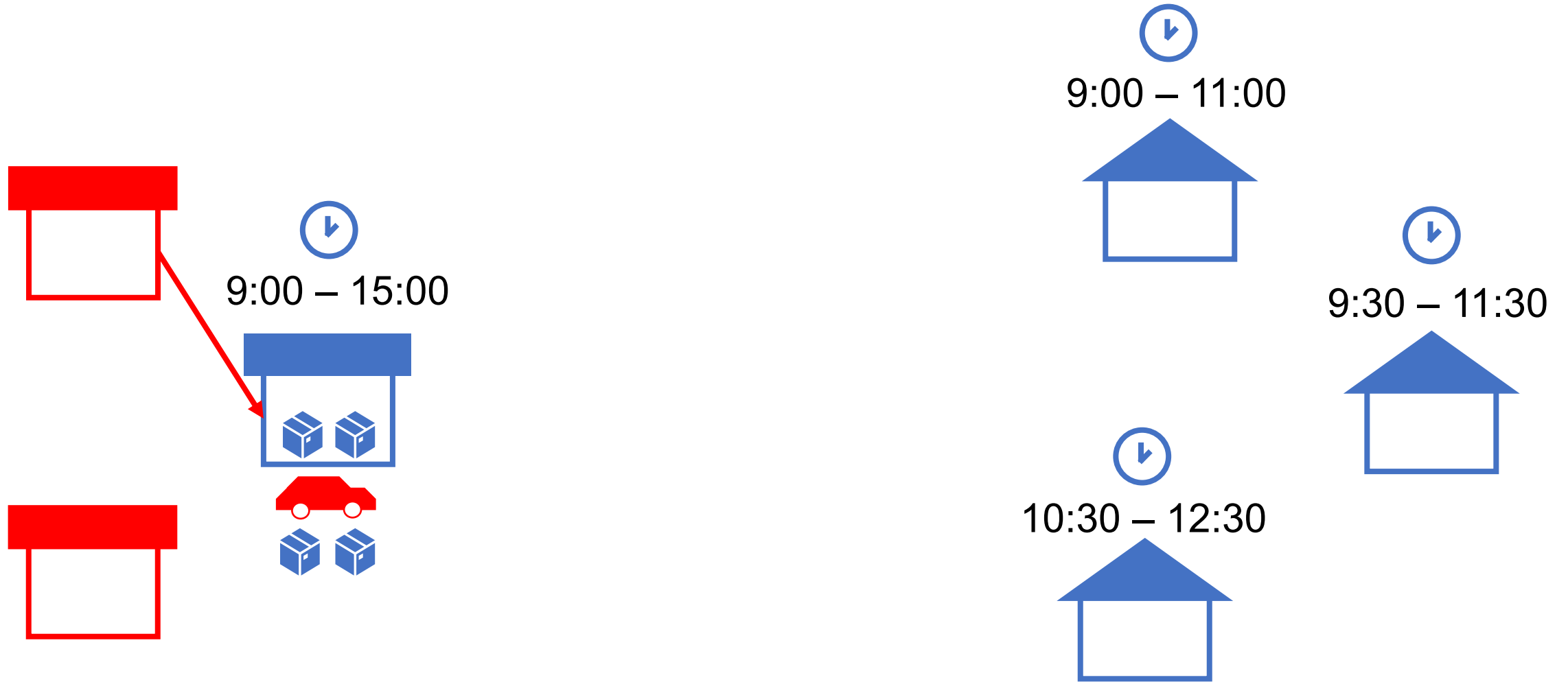


# Problem Definition

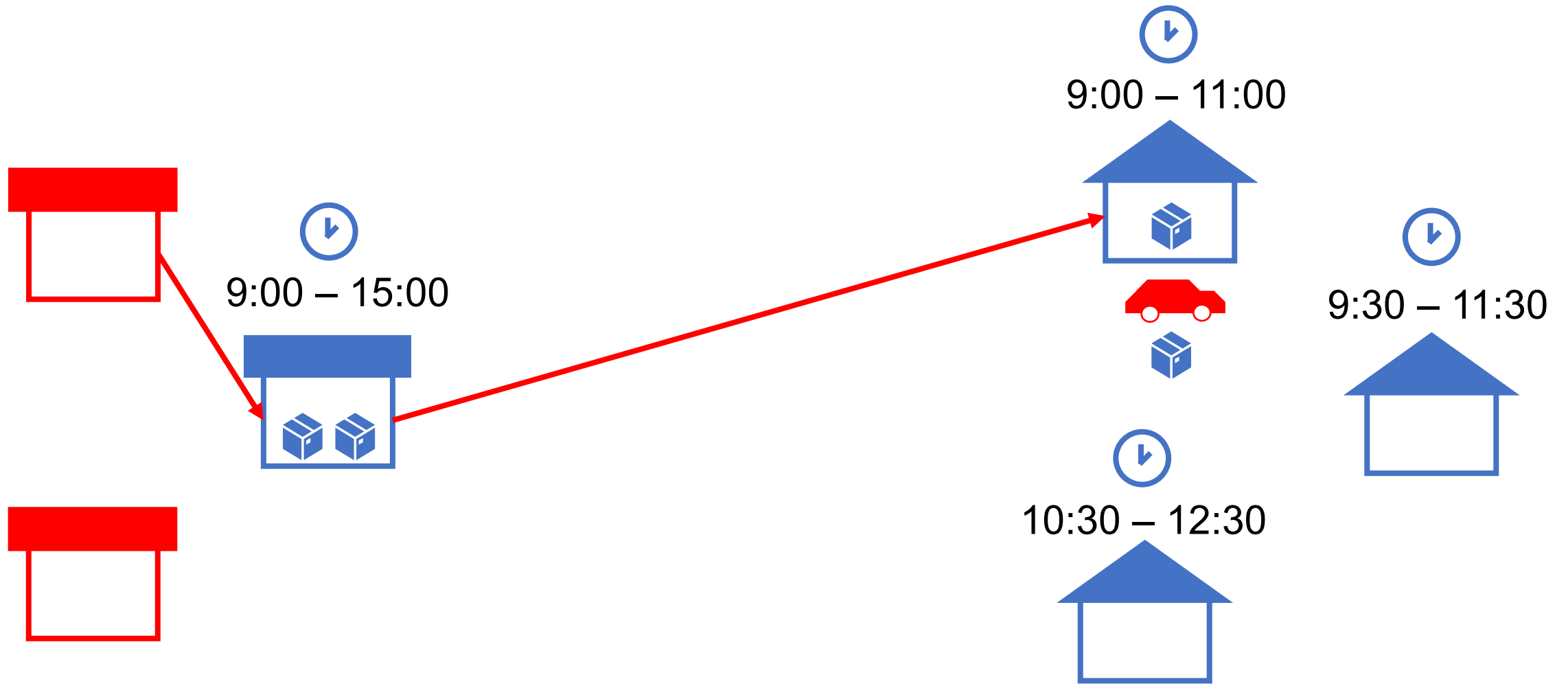
# Pickup at the Center and Deliver to Customers



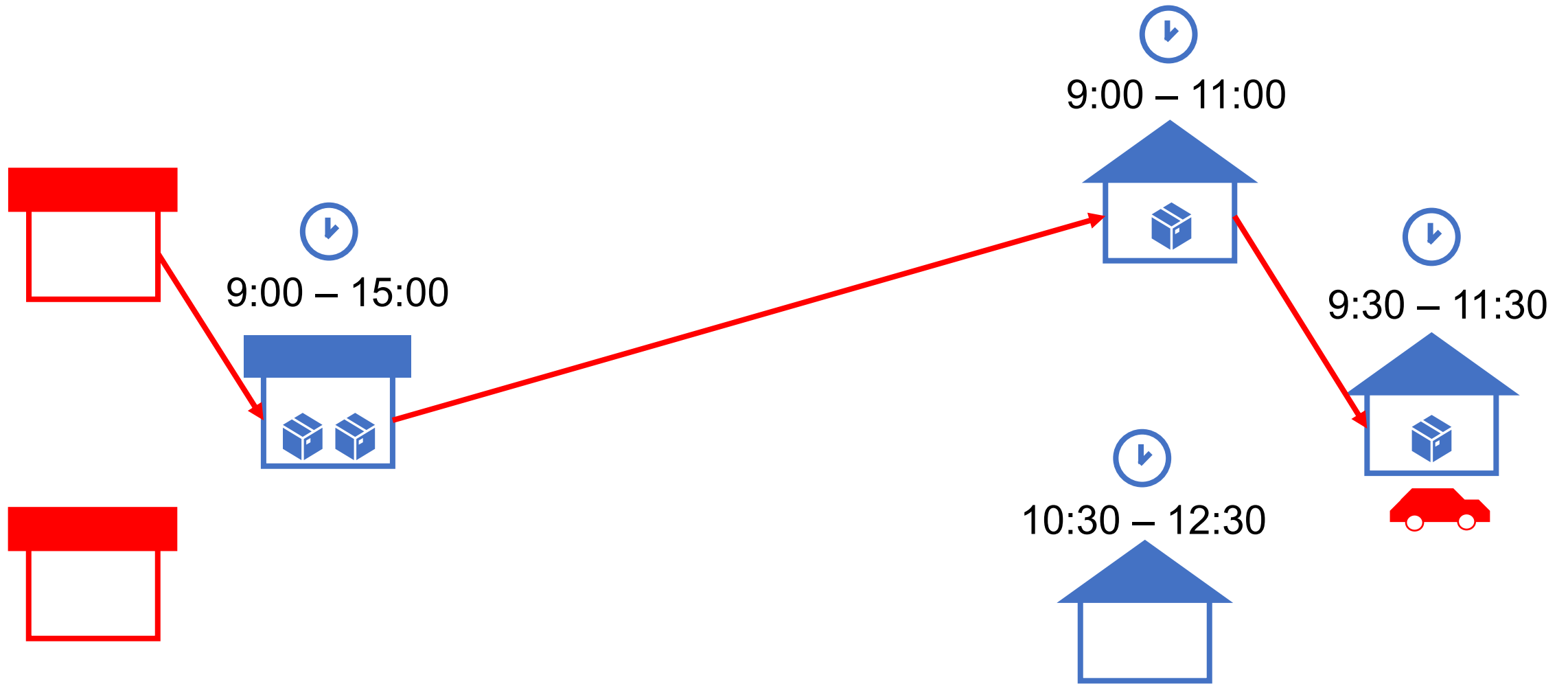
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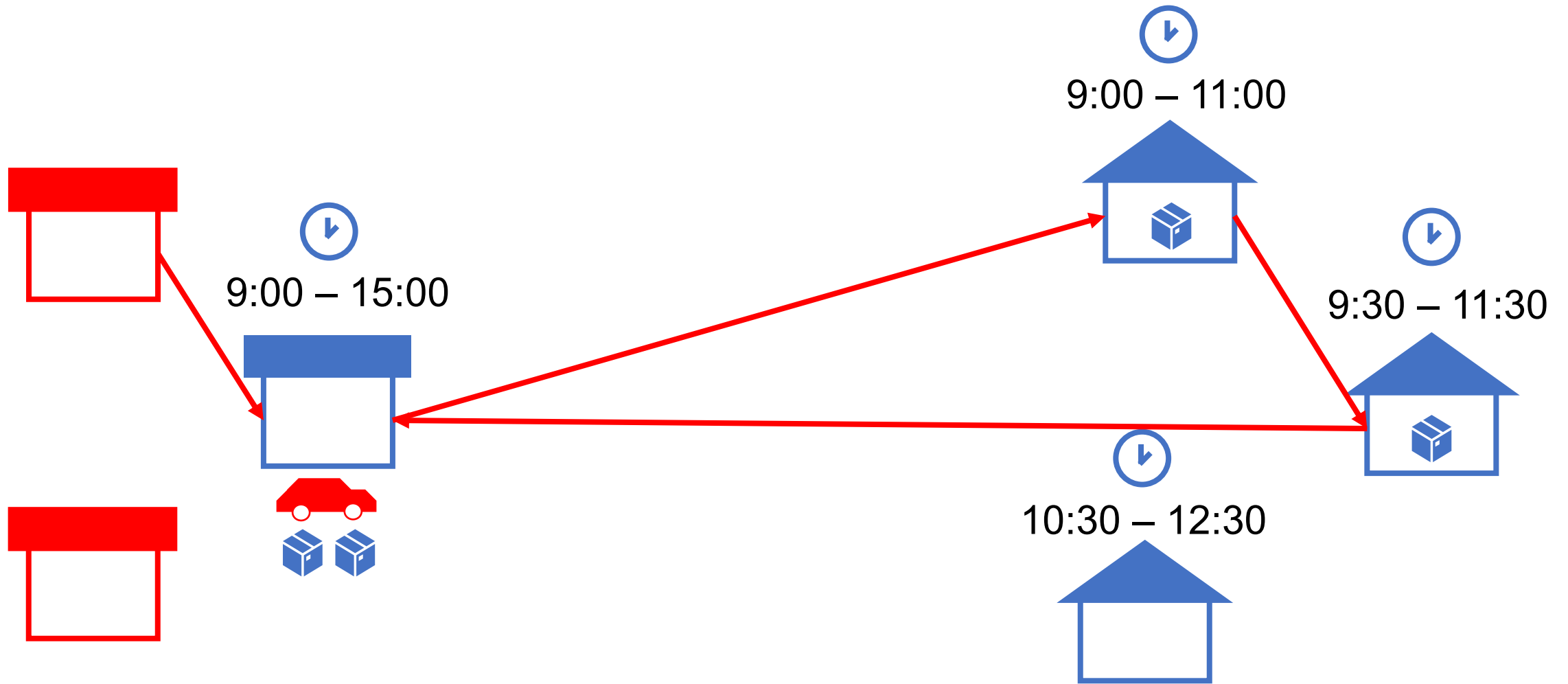
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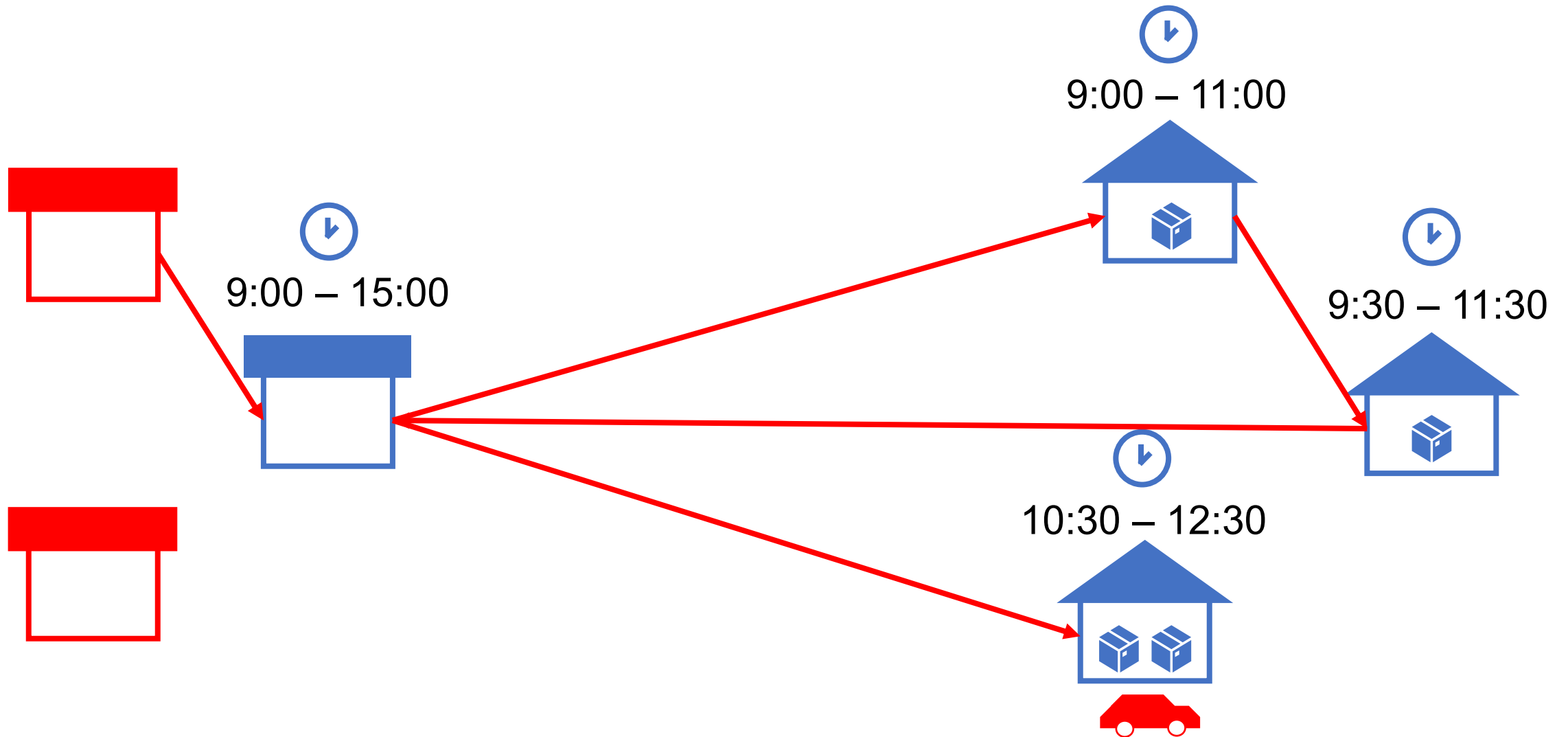
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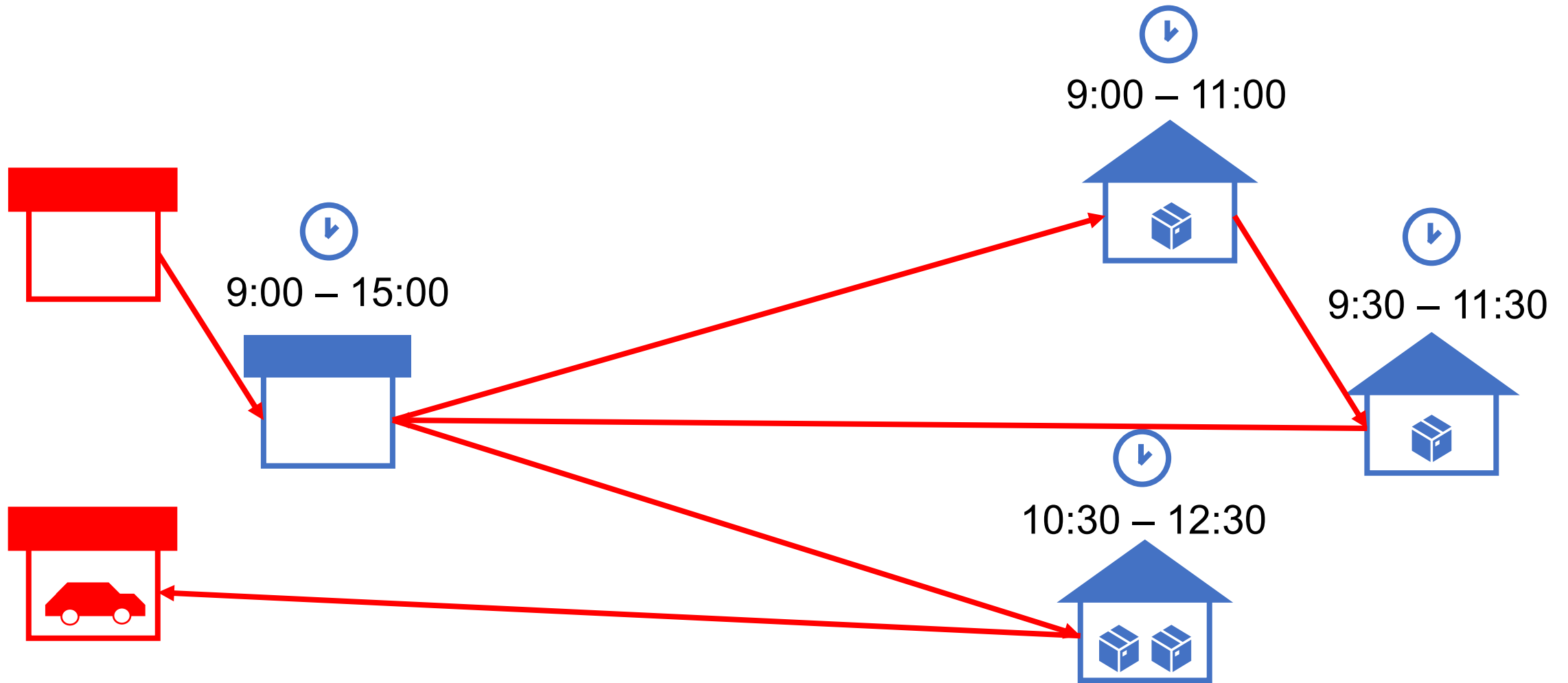
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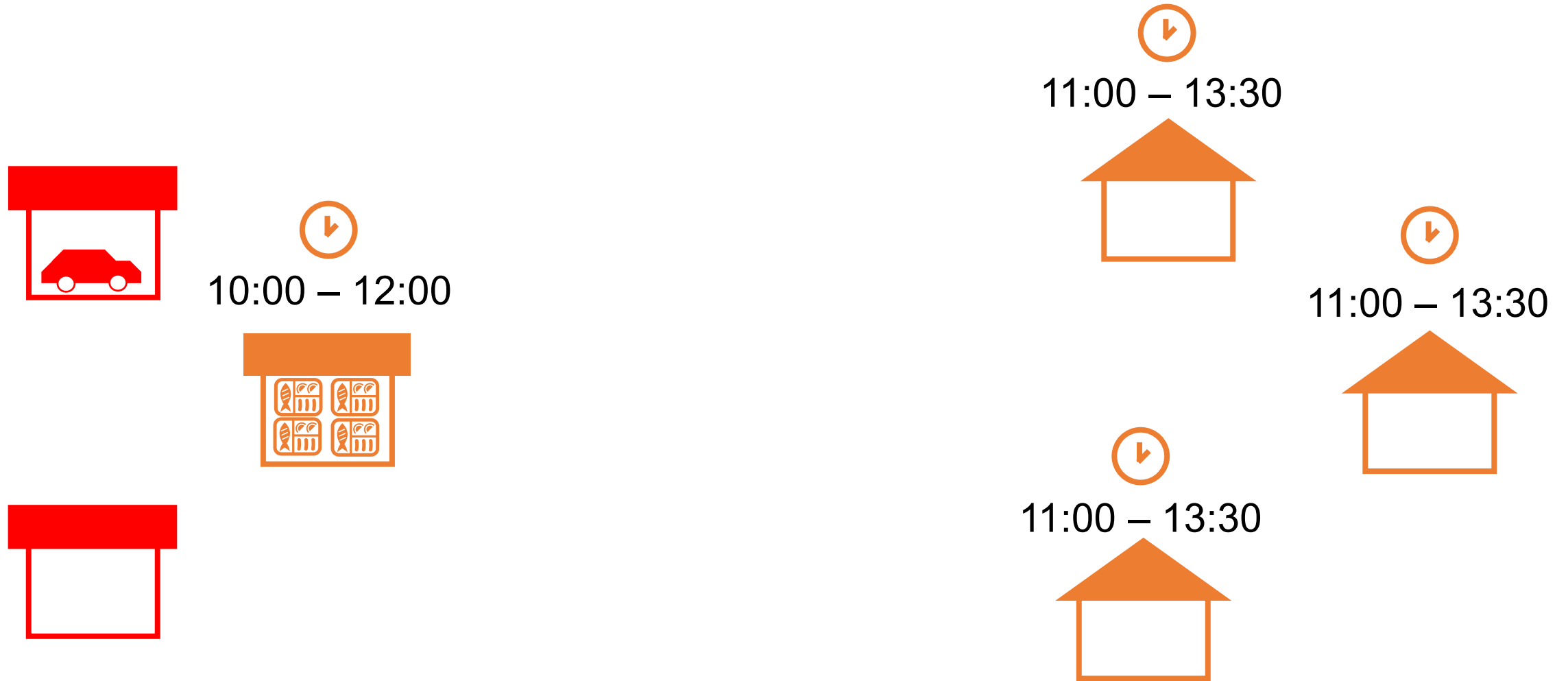
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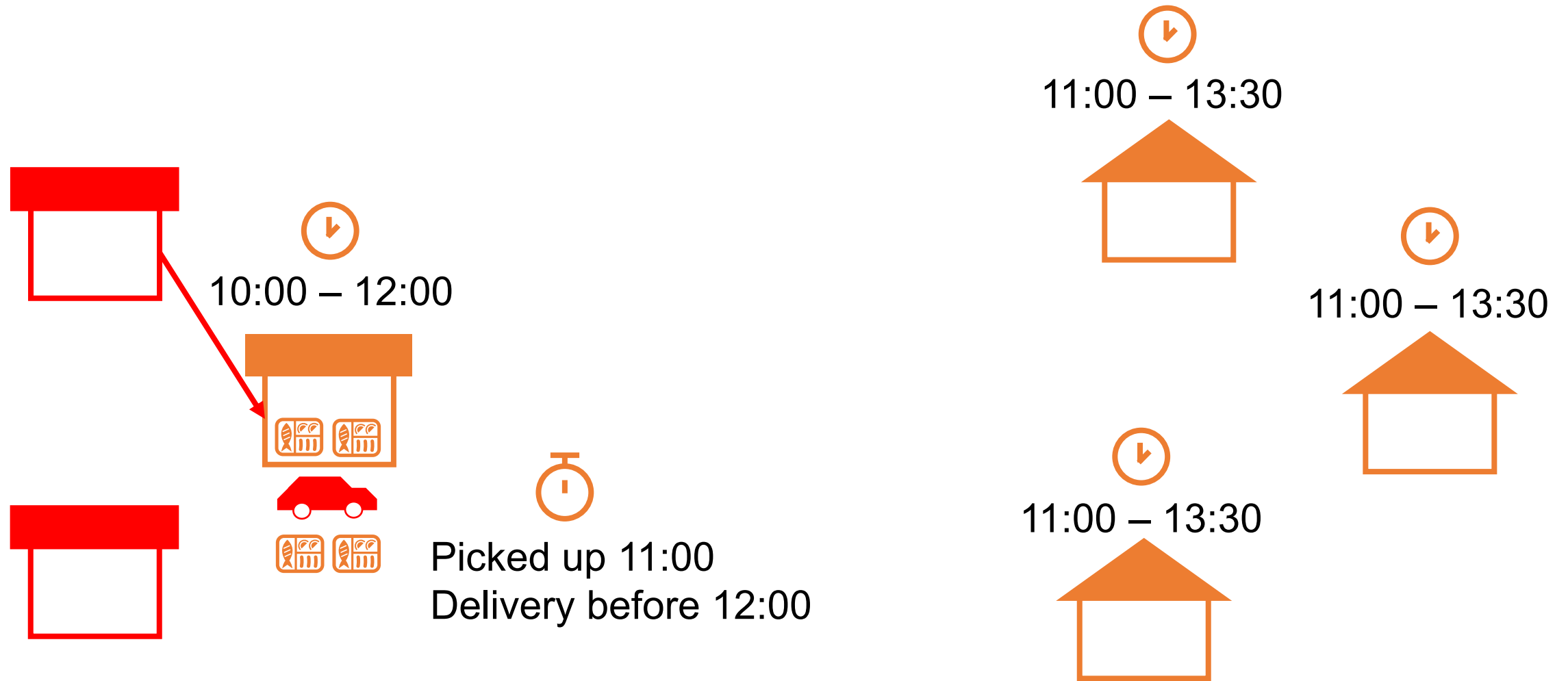
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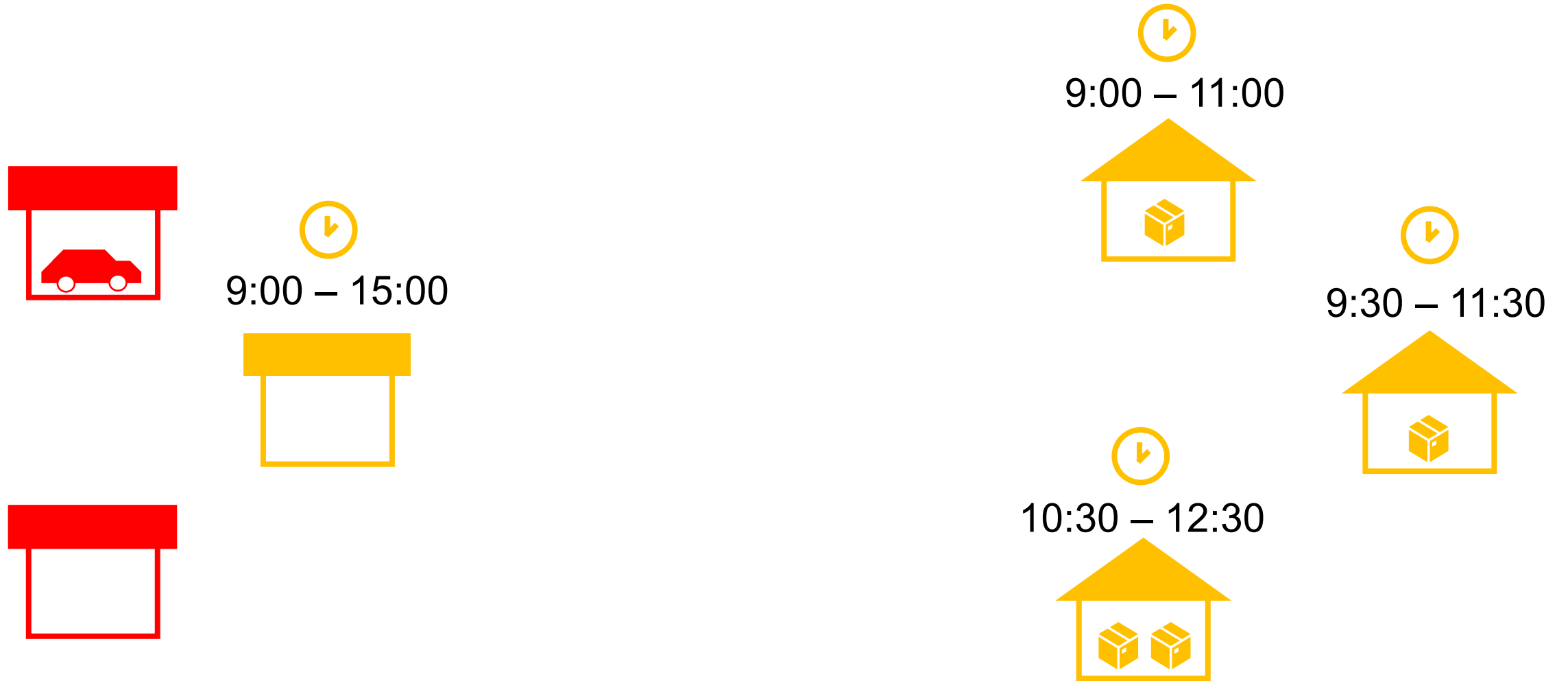
# Maximum Transportation Time for Perishable Goods



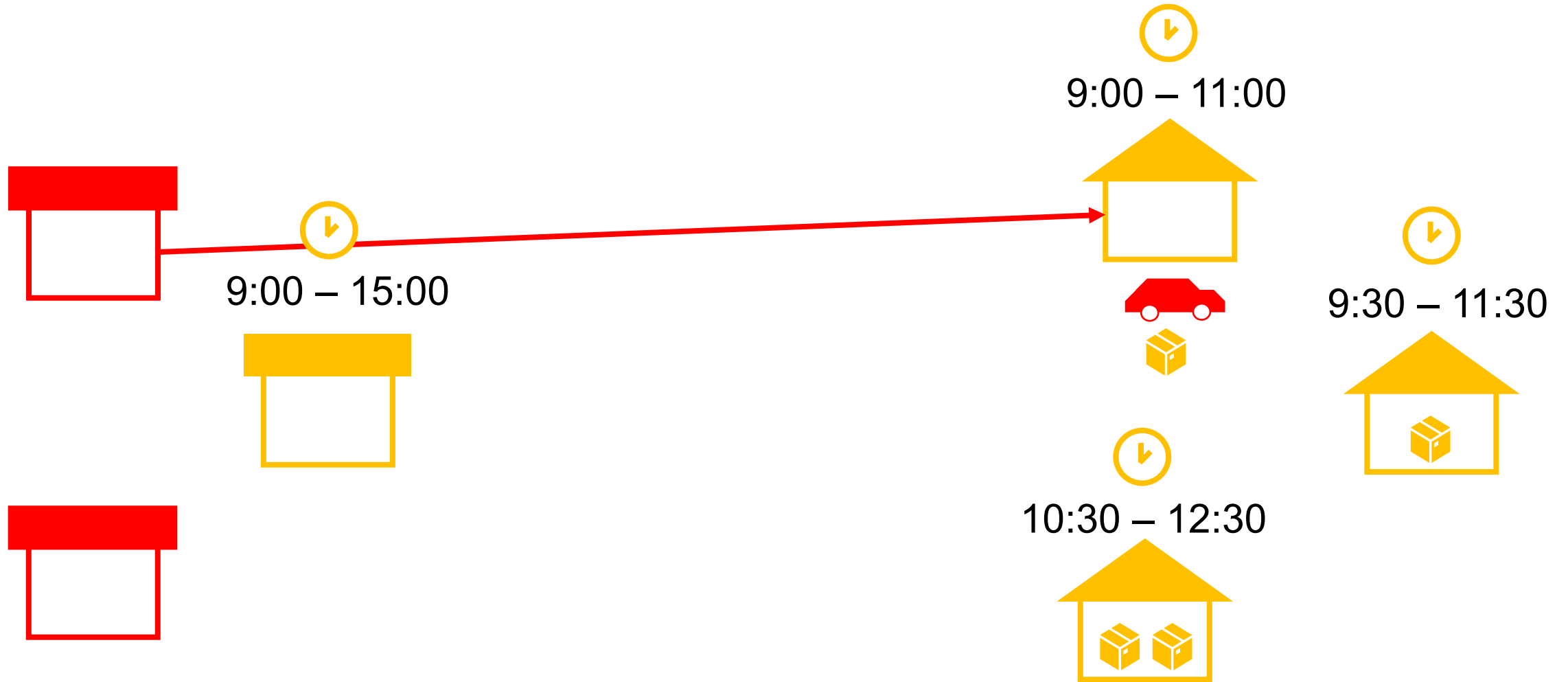
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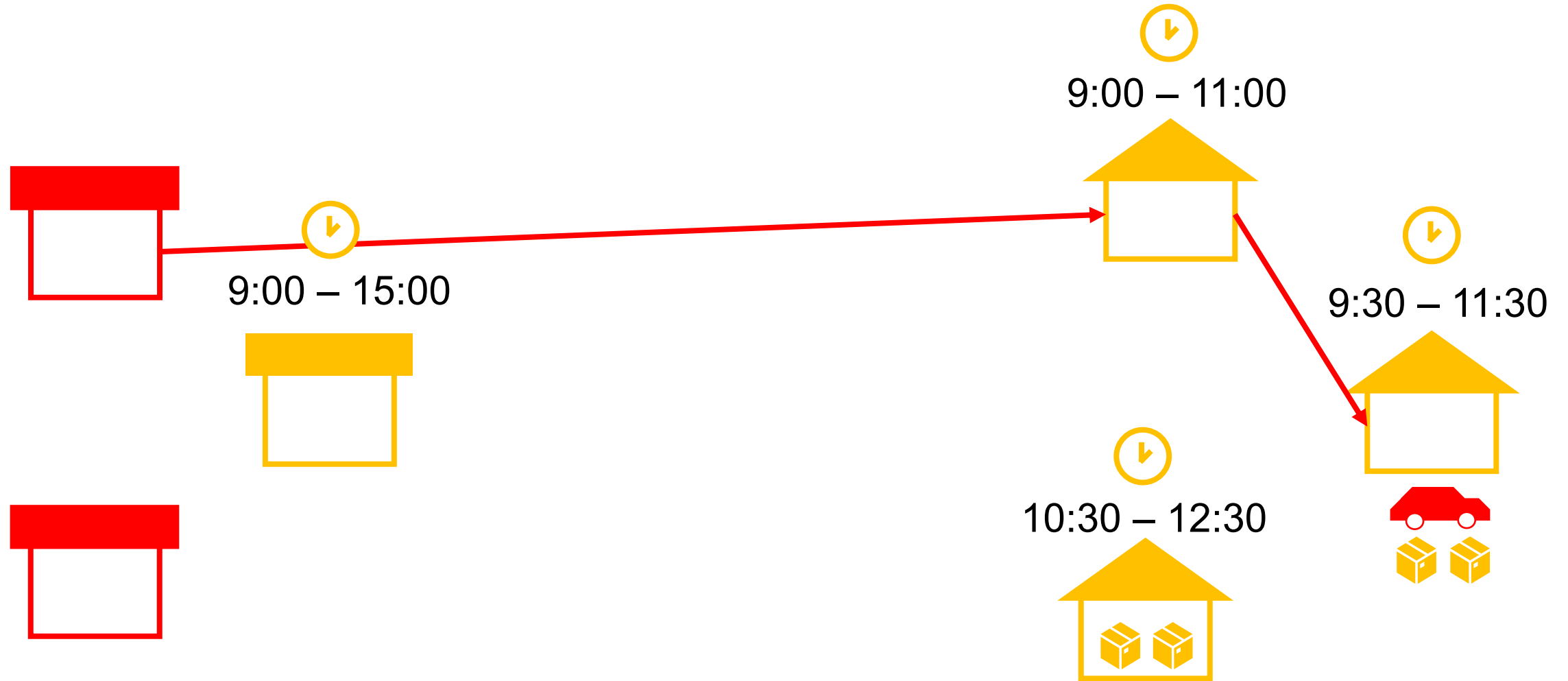
# Pickup at Customers and Deliver to the Center



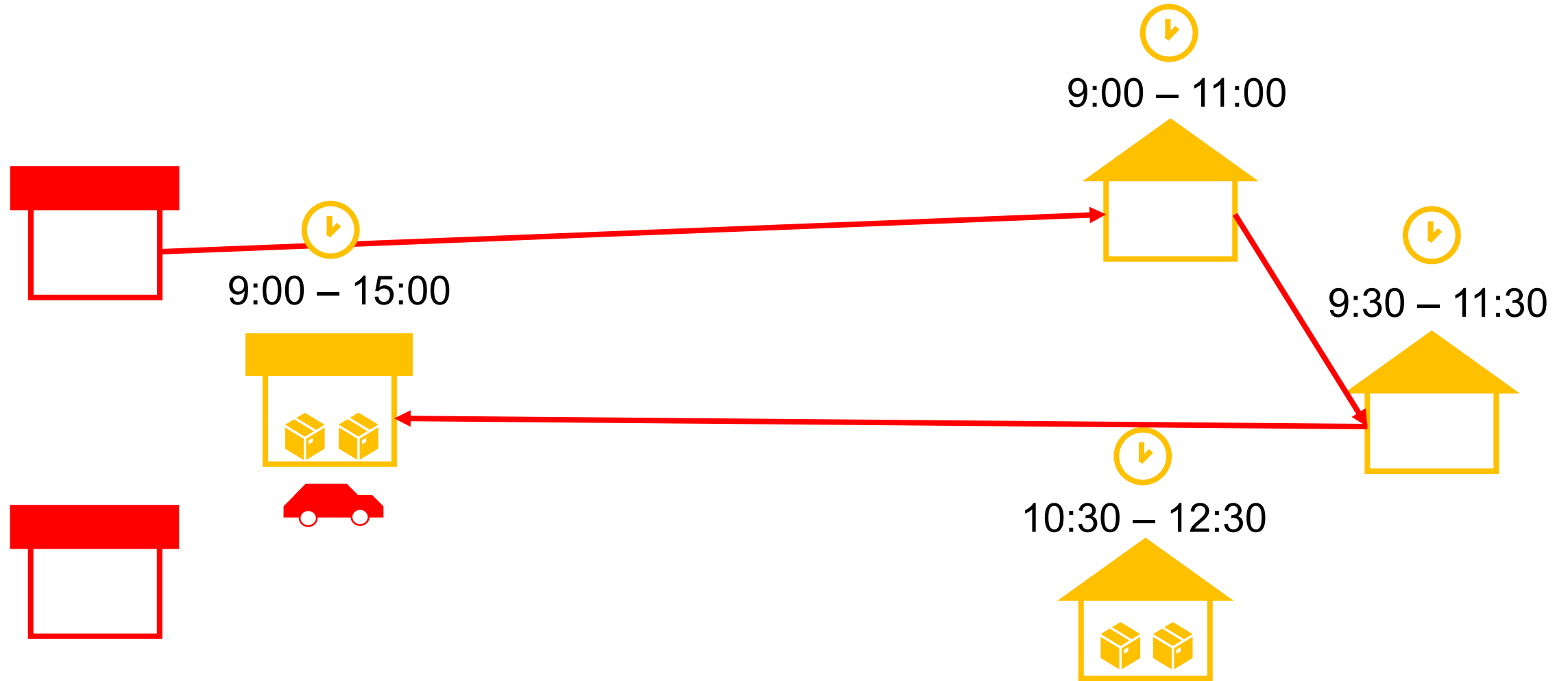
# Pickup at Customers and Deliver to the Center



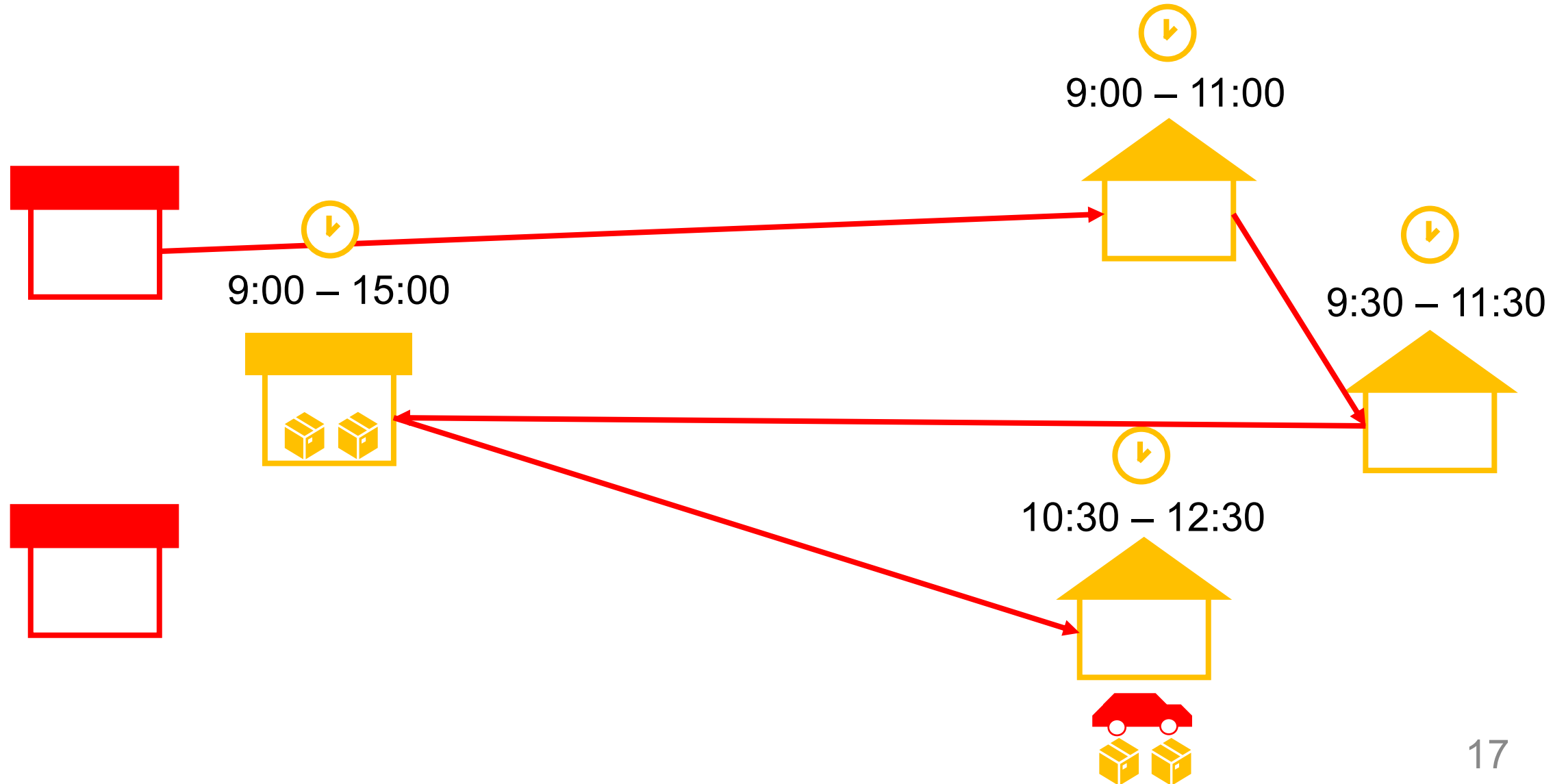
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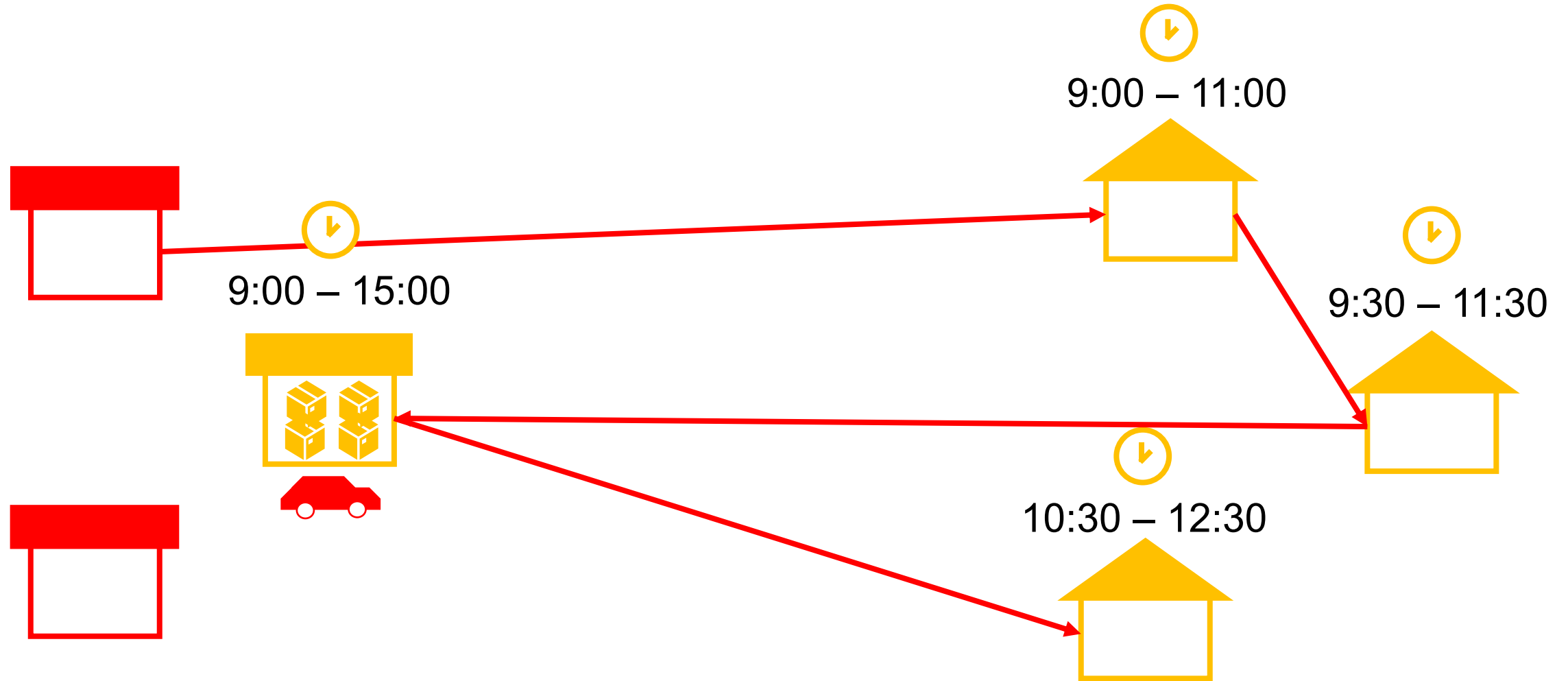
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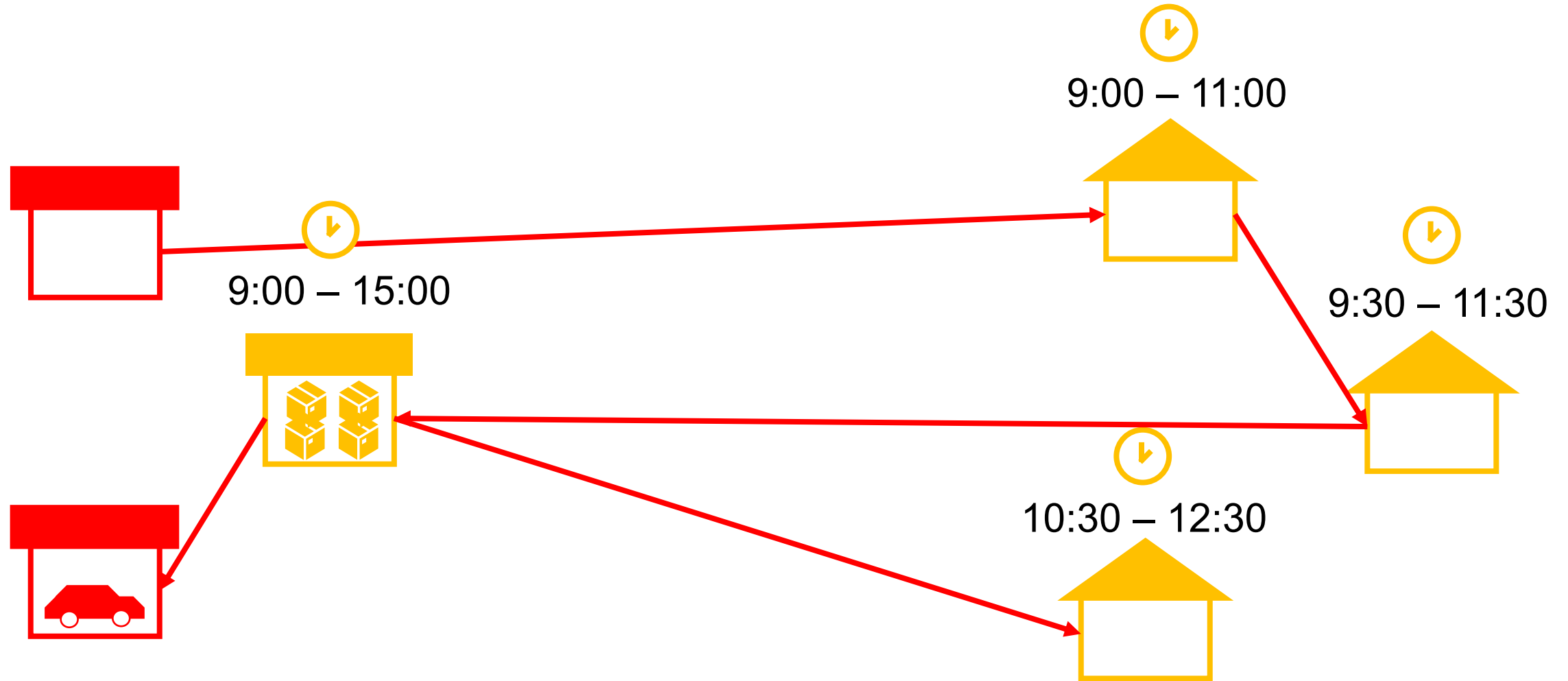
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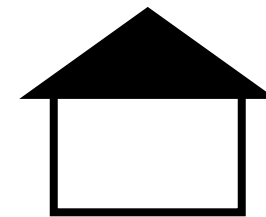
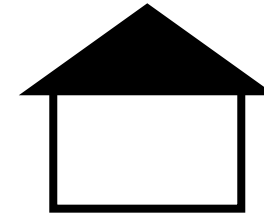
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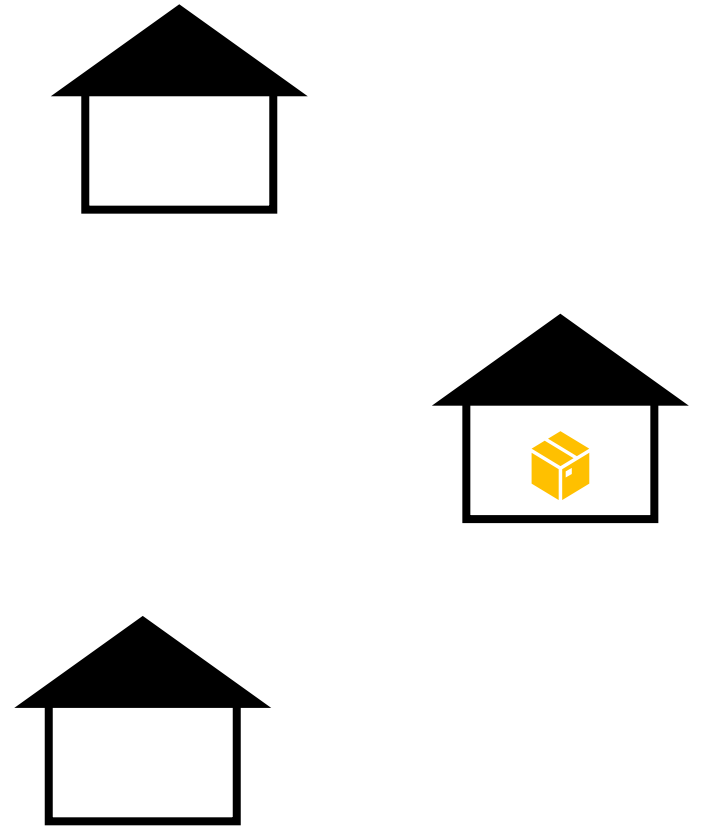
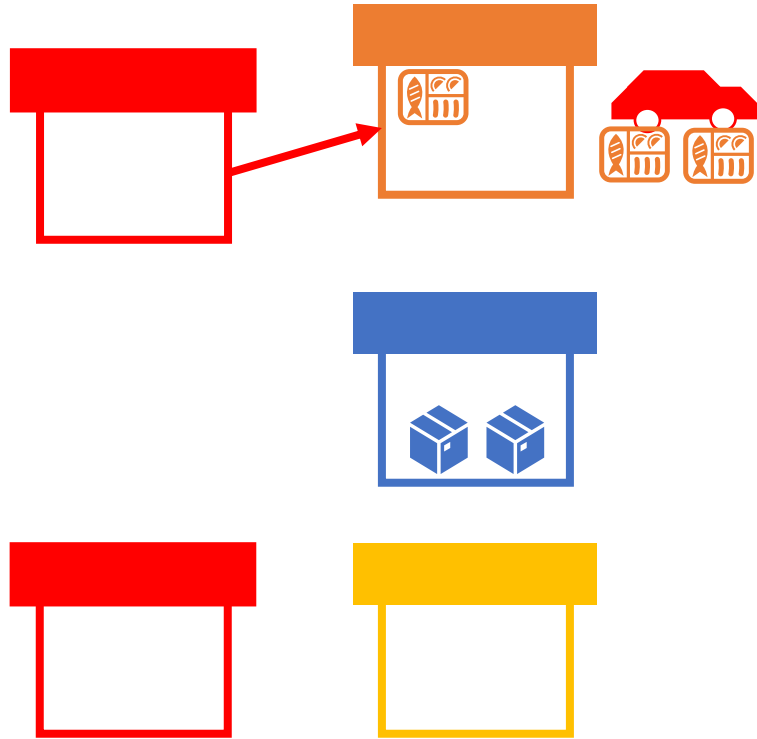
# Pickup at Customers and Deliver to the Center



# Multi-Commodity for a Single Vehicle



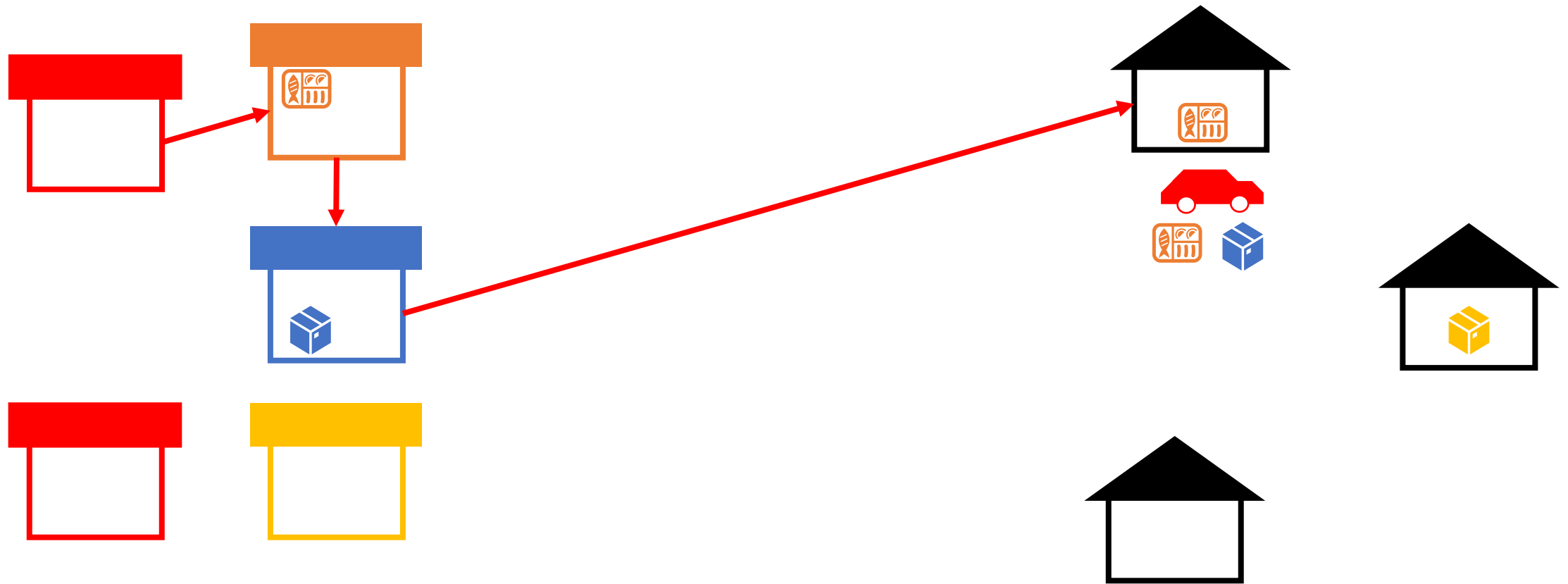
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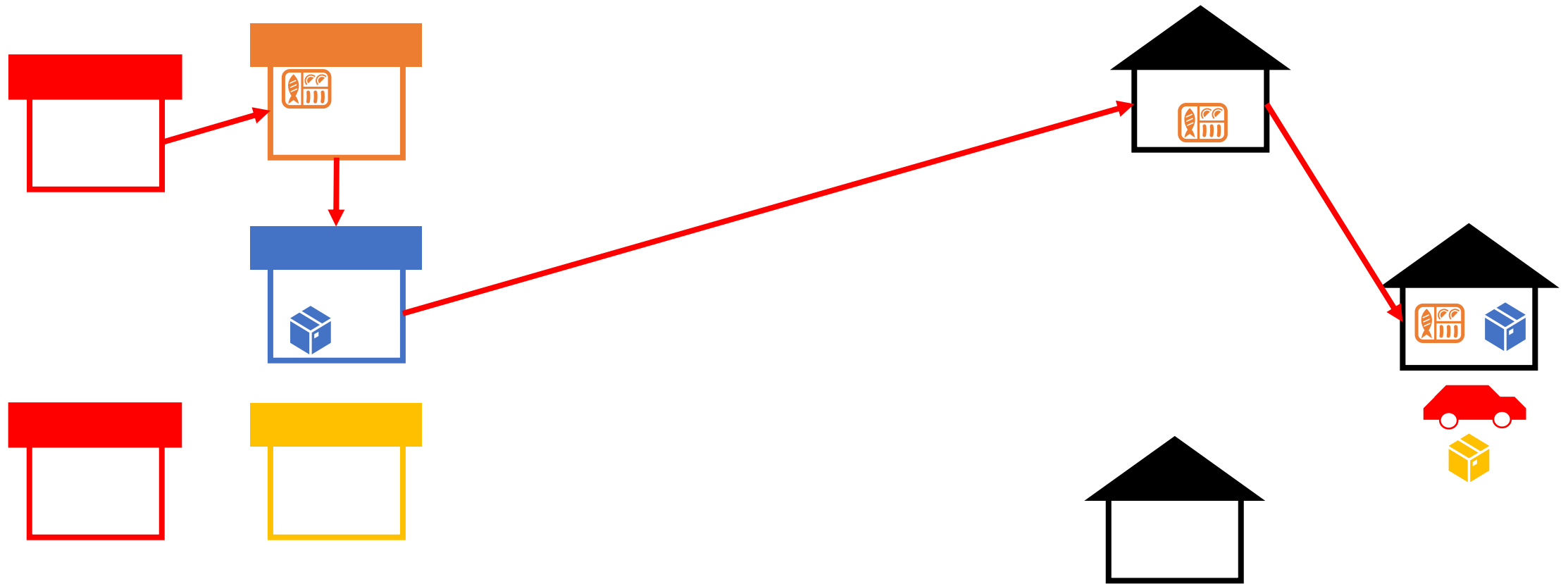
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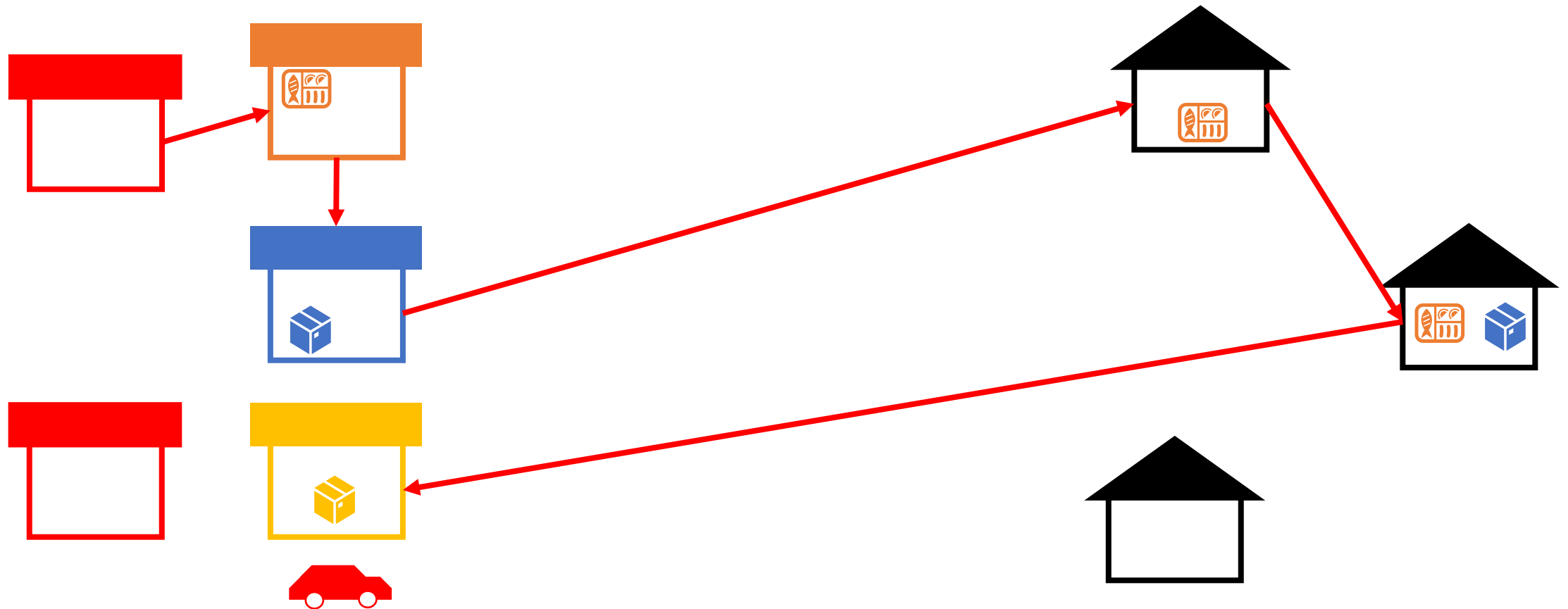
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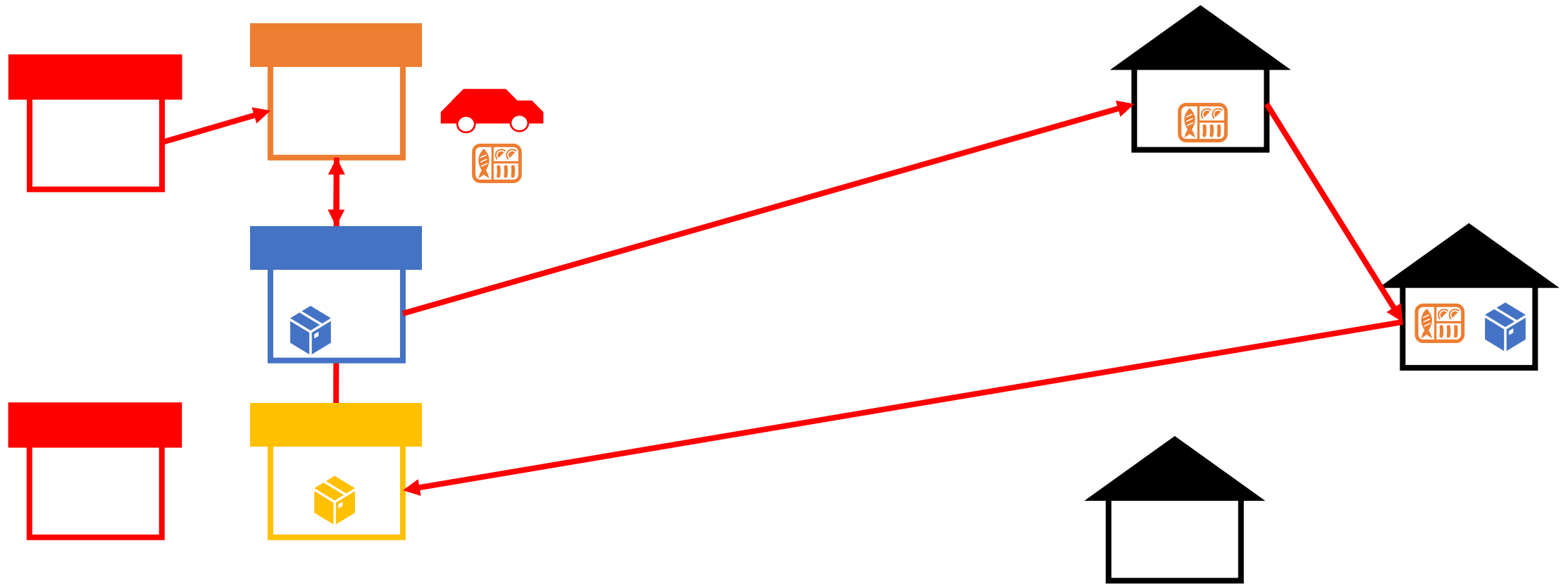
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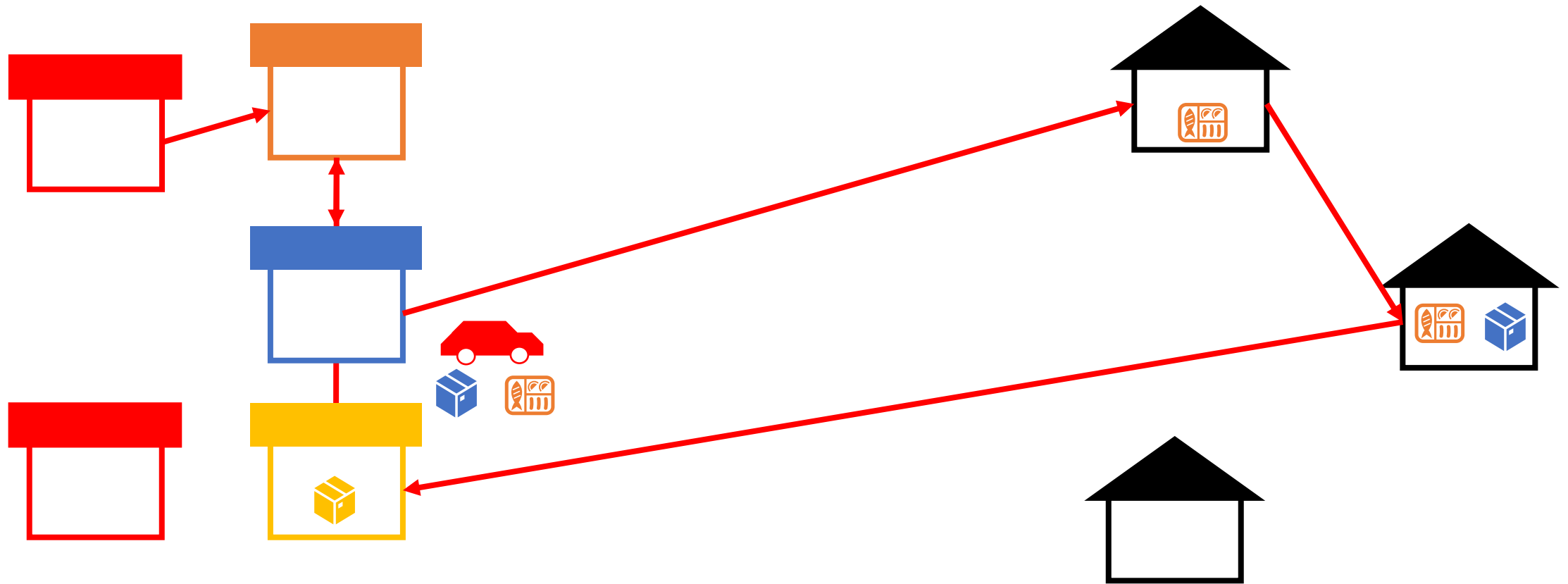
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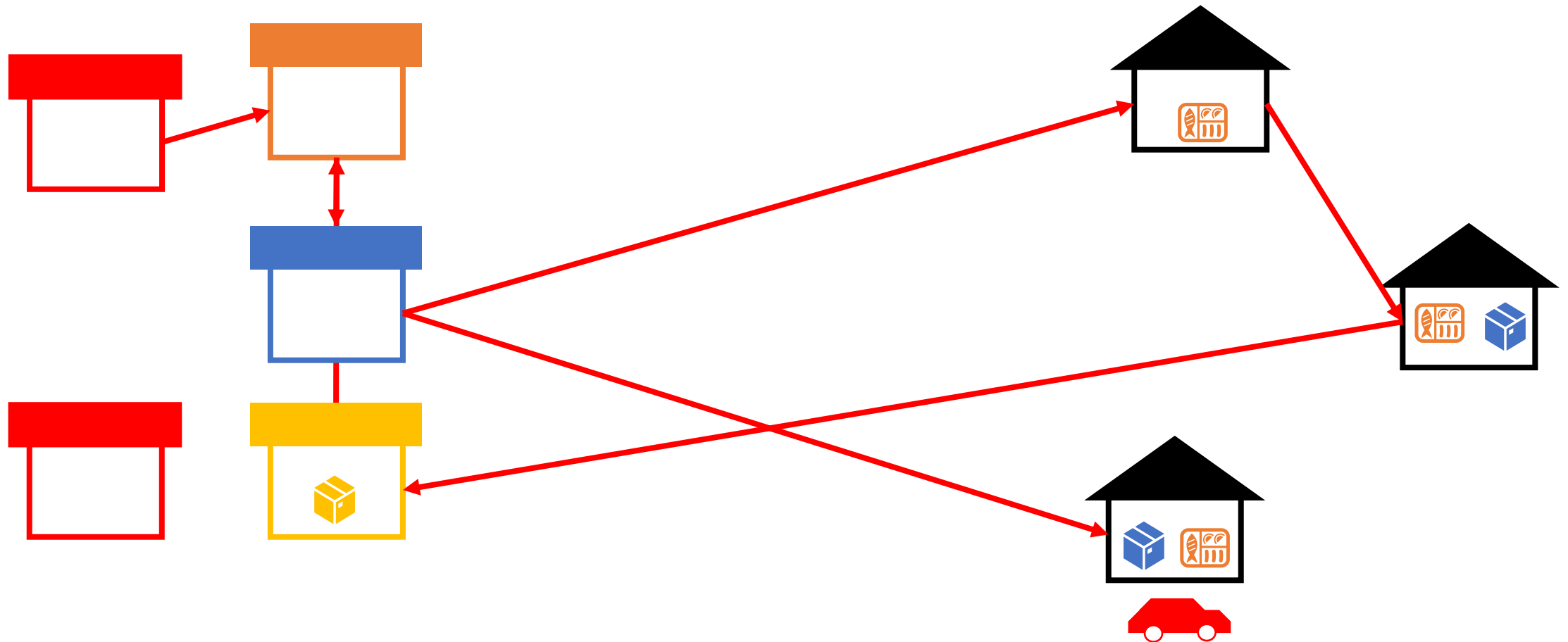
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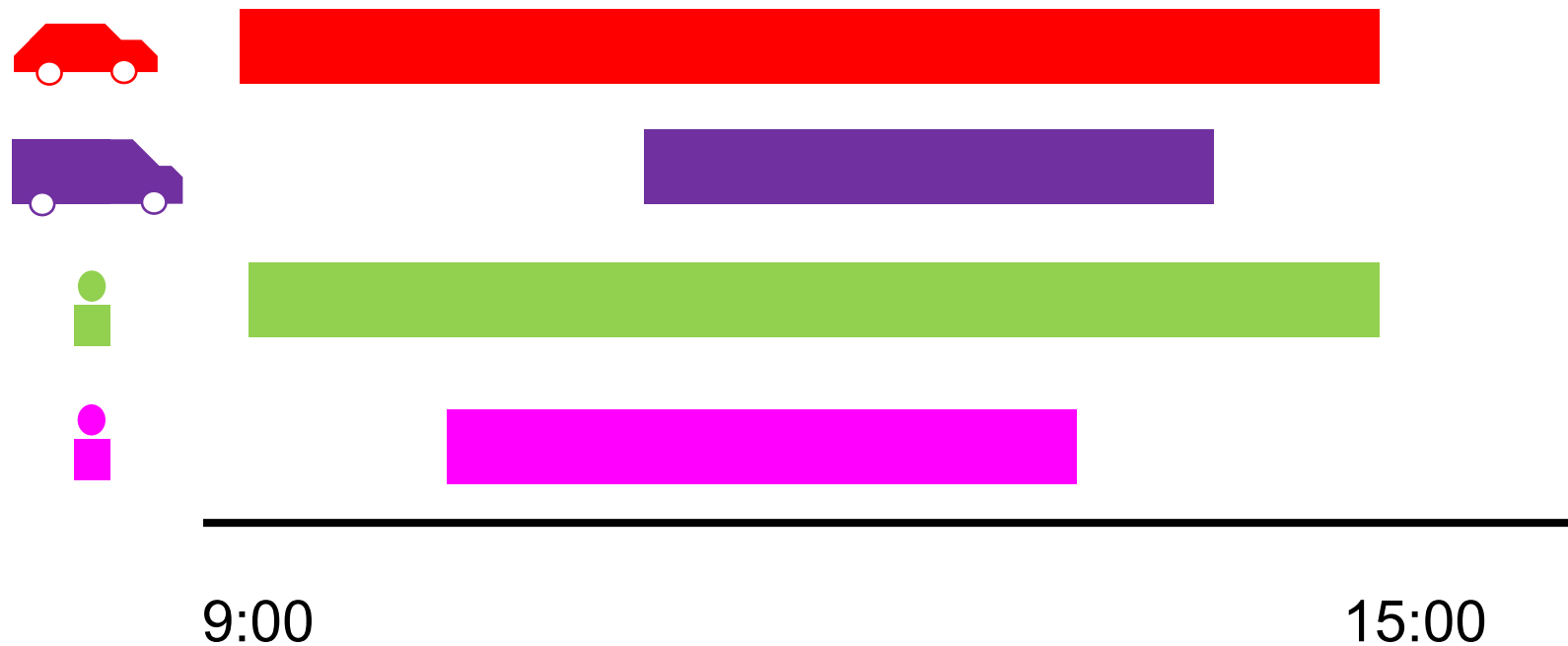


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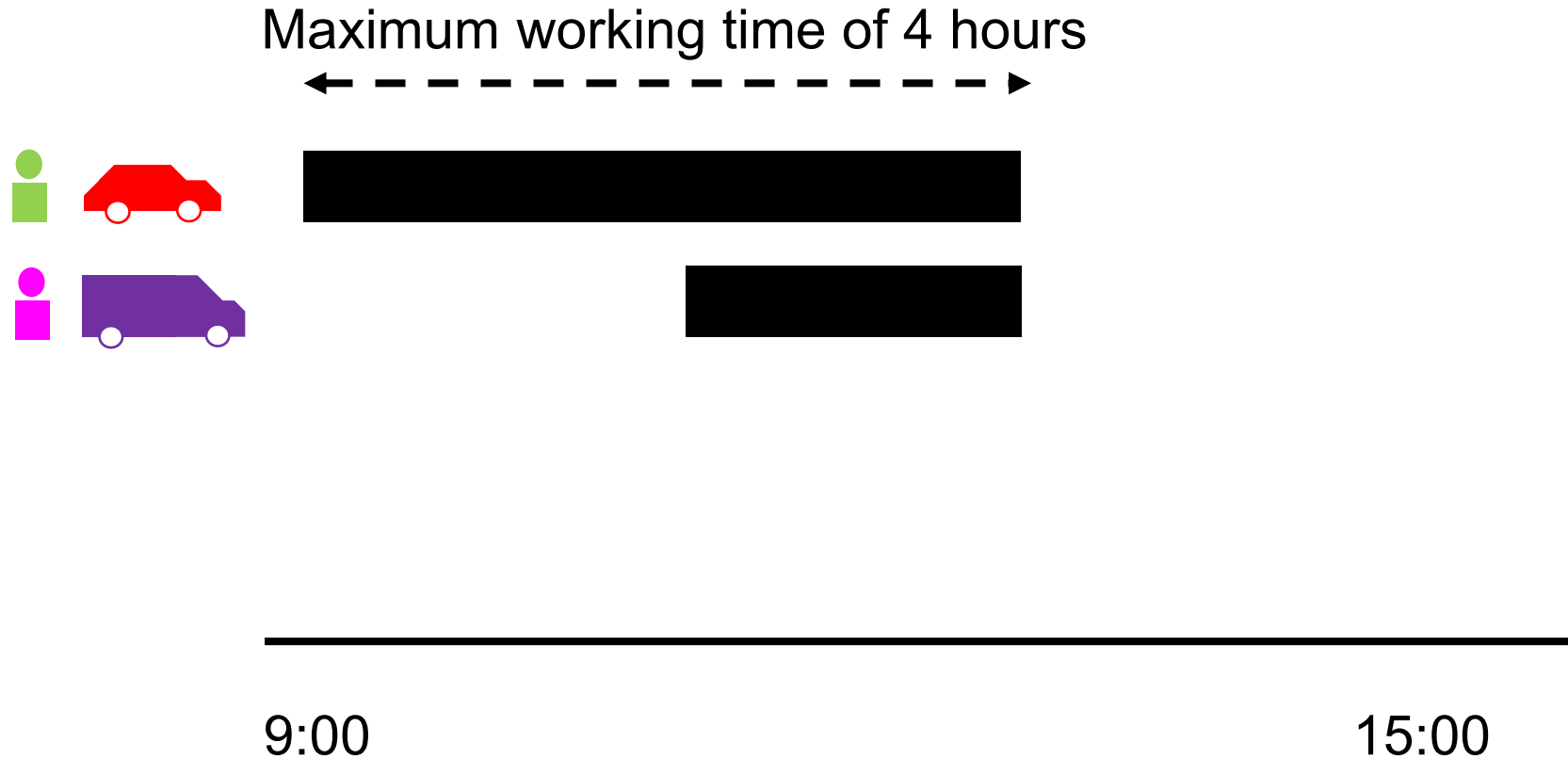




# Heterogeneous Vehicles and Driver Assignments



# Heterogeneous Vehicles and Driver Assignments



# Objective

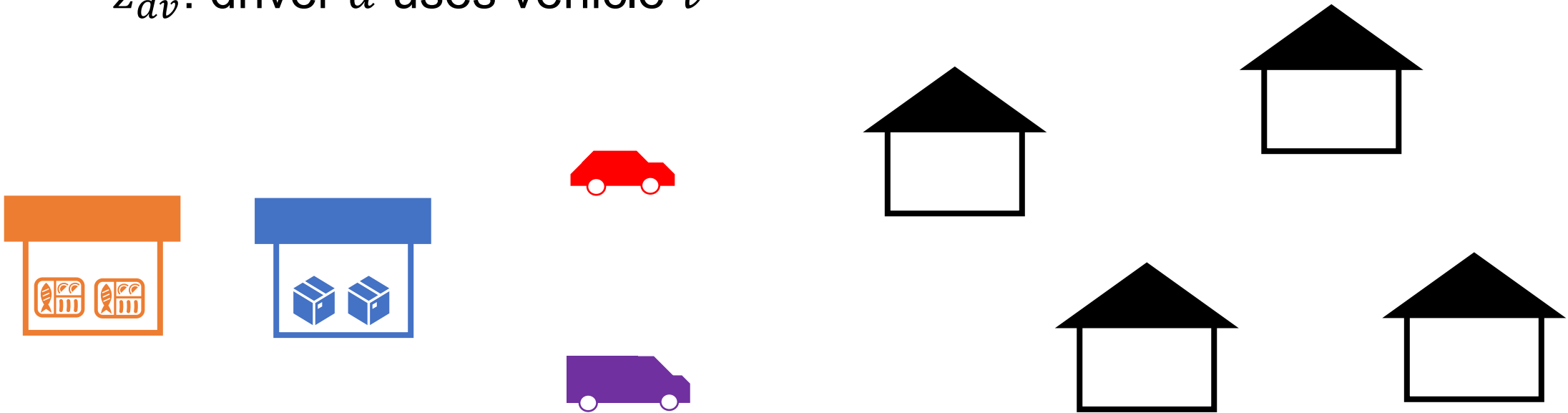
Maximize the weighted number of served requests



# Constraint Programming Model

# Main Decision Variables

- $x_{ij}$ : location  $j$  is visited from location  $i$   
Treat multiple requests by the same customer as different locations  
Duplicate the center for multiple visits by each vehicle
- $y_{rv}$ : request  $r$  is served by vehicle  $v$
- $z_{dv}$ : driver  $d$  uses vehicle  $v$



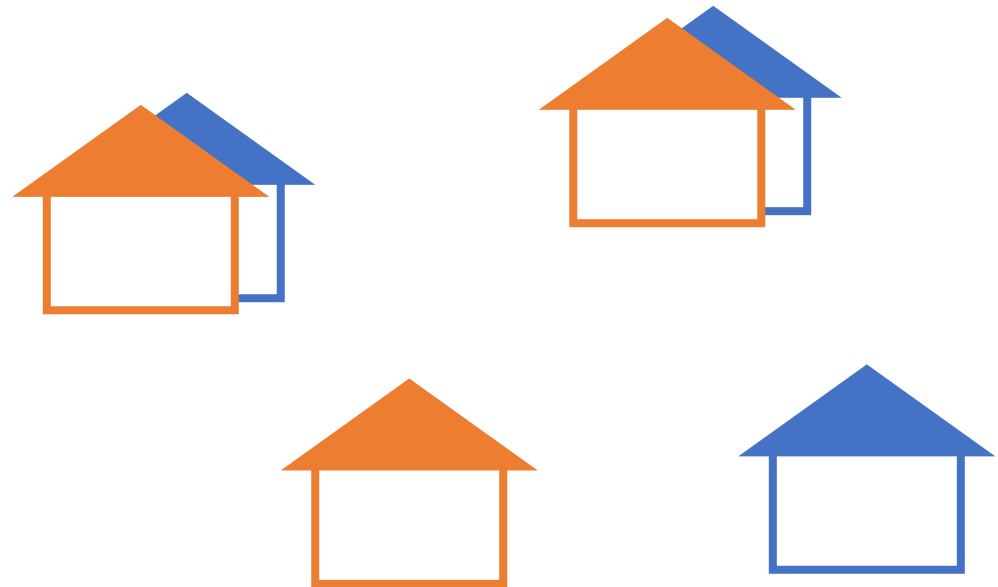
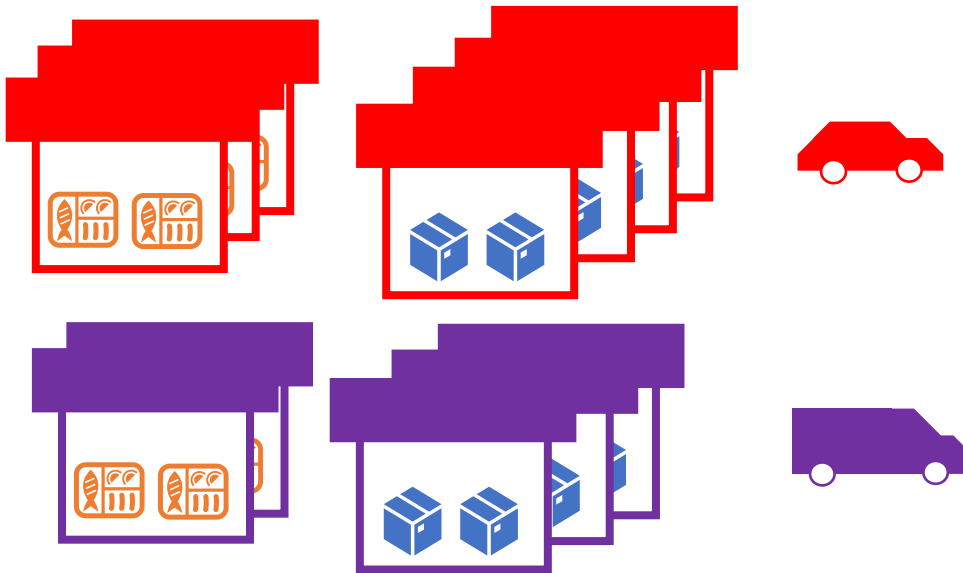
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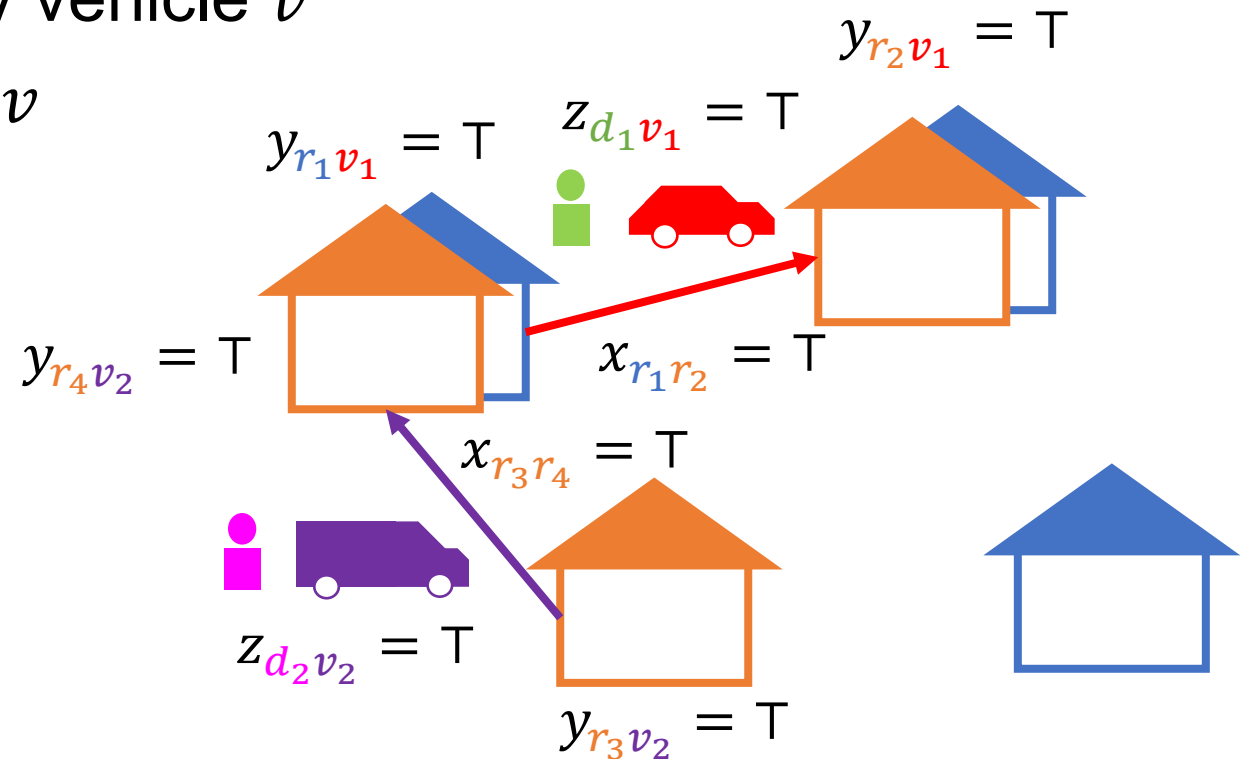
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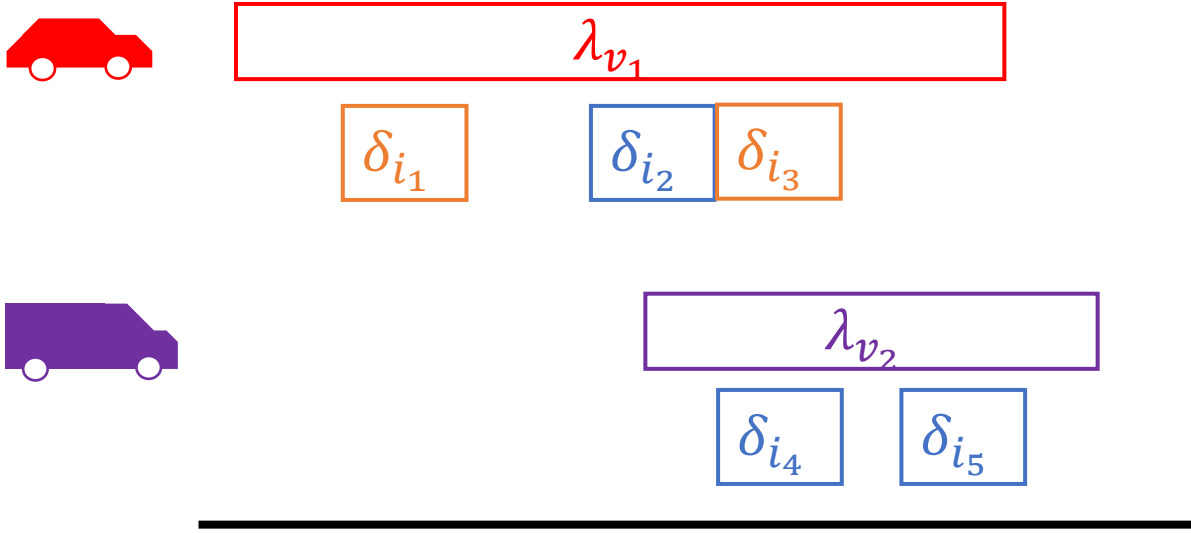
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# Travel Time Variables

- $\lambda_v$ : optional interval representing the tour duration of vehicle  $v$
- $\delta_i$ : optional fixed-length interval representing the visit to location  $i$
- Objective:  $\max \sum_{r \in \mathcal{R}} p_r \mathbb{1}(\text{Pres}(\delta_r))$  where  $p_r$  is the score of  $r$



$$\text{AtMostOne}(\{x_{vj} \mid (v, j) \in \mathcal{A}\}) \quad \forall v \in \mathcal{V} \quad (2)$$

$$\text{AtMostOne}(\{x_{iv} \mid (i, v) \in \mathcal{A}\}) \quad \forall v \in \mathcal{V} \quad (3)$$

$$\text{AtMostOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\}) \quad \forall j \in \mathcal{R} \cup \mathcal{O} \quad (4)$$

$$\text{AtMostOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\}) \quad \forall i \in \mathcal{R} \cup \mathcal{O} \quad (5)$$

$$x_{vj} \implies \text{Pres}(\lambda_v) \wedge \text{Pres}(\delta_j) \quad \forall (v, j) \in \mathcal{A}, v \in \mathcal{V} \quad (6)$$

$$x_{iv} \implies \text{Pres}(\delta_i) \wedge \text{Pres}(\lambda_v) \quad \forall (i, v) \in \mathcal{A}, v \in \mathcal{V} \quad (7)$$

$$x_{ij} \implies \text{Pres}(\delta_i) \wedge \text{Pres}(\delta_j) \quad \forall (i, j) \in \mathcal{A}, i, j \notin \mathcal{V} \quad (8)$$

$$\text{Pres}(\lambda_v) \implies \text{AtLeastOne}(\{x_{vj} \mid (v, j) \in \mathcal{A}\}) \quad \forall v \in \mathcal{V} \quad (9)$$

$$\text{Pres}(\lambda_v) \implies \text{AtLeastOne}(\{x_{iv} \mid (i, v) \in \mathcal{A}\}) \quad \forall v \in \mathcal{V} \quad (10)$$

$$\text{Pres}(\delta_j) \implies \text{AtLeastOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\}) \quad \forall j \in \mathcal{R} \cup \mathcal{O} \quad (11)$$

$$\text{Pres}(\delta_i) \implies \text{AtLeastOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\}) \quad \forall i \in \mathcal{R} \cup \mathcal{O} \quad (12)$$

$$x_{vj} \implies \text{Start}(\delta_j) \geq \text{Start}(\lambda_v) + c_{vj} \quad \forall (v, j) \in \mathcal{A}, v \in \mathcal{V} \quad (13)$$

$$x_{iv} \implies \text{End}(\lambda_v) \geq \text{End}(\delta_i) + c_{iv} \quad \forall (i, v) \in \mathcal{A}, v \in \mathcal{V} \quad (14)$$

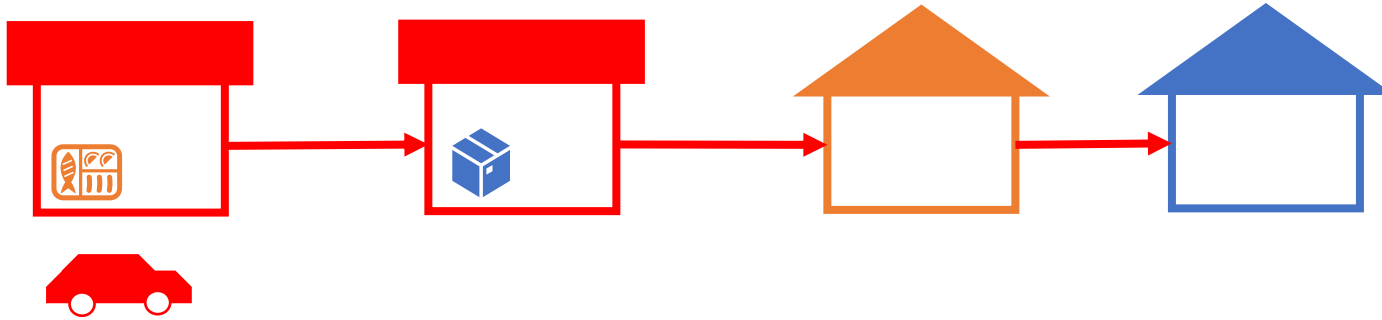
$$x_{ij} \implies \text{Start}(\delta_j) \geq \text{End}(\delta_i) + c_{ij} \quad \forall (i, j) \in \mathcal{A}, i, j \notin \mathcal{V} \quad (15)$$

# Travel Time Variables

- $\lambda_v$ : optional interval representing the tour duration of vehicle  $v$
  - $\delta_i$ : optional fixed-length interval representing the visit to location  $i$
  - Objective:  $\max \sum_{r \in \mathcal{R}} p_r \mathbb{1}(\text{Pres}(\delta_r))$  where  $p_r$  is the score of  $r$
  - NoOverlap is not used as CP-SAT does not support a distance matrix
  - Circuit could be used with  $x_{ij}$  in CP-SAT but has worse performance
- |                                                                                             |                                                           |      |
|---------------------------------------------------------------------------------------------|-----------------------------------------------------------|------|
| $\text{AtMostOne}(\{x_{vj} \mid (v, j) \in \mathcal{A}\})$                                  | $\forall v \in \mathcal{V}$                               | (2)  |
| $\text{AtMostOne}(\{x_{iv} \mid (i, v) \in \mathcal{A}\})$                                  | $\forall v \in \mathcal{V}$                               | (3)  |
| $\text{AtMostOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\})$                                  | $\forall j \in \mathcal{R} \cup \mathcal{O}$              | (4)  |
| $\text{AtMostOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\})$                                  | $\forall i \in \mathcal{R} \cup \mathcal{O}$              | (5)  |
| $x_{vj} \implies \text{Pres}(\lambda_v) \wedge \text{Pres}(\delta_j)$                       | $\forall (v, j) \in \mathcal{A}, v \in \mathcal{V}$       | (6)  |
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| $\text{Pres}(\delta_j) \implies \text{AtLeastOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\})$  | $\forall j \in \mathcal{R} \cup \mathcal{O}$              | (11) |
| $\text{Pres}(\delta_i) \implies \text{AtLeastOne}(\{x_{ij} \mid (i, j) \in \mathcal{A}\})$  | $\forall i \in \mathcal{R} \cup \mathcal{O}$              | (12) |
| $x_{vj} \implies \text{Start}(\delta_j) \geq \text{Start}(\lambda_v) + c_{vj}$              | $\forall (v, j) \in \mathcal{A}, v \in \mathcal{V}$       | (13) |
| $x_{iv} \implies \text{End}(\lambda_v) \geq \text{End}(\delta_i) + c_{iv}$                  | $\forall (i, v) \in \mathcal{A}, v \in \mathcal{V}$       | (14) |
| $x_{ij} \implies \text{Start}(\delta_j) \geq \text{End}(\delta_i) + c_{ij}$                 | $\forall (i, j) \in \mathcal{A}, i, j \notin \mathcal{V}$ | (15) |

# Load and Transportation Time Variables

- $l_{gi}$ : load of commodity  $g$  when arriving at location  $i$
- $\tau_{gi}$ : transportation time of commodity  $g$  when arriving at location  $i$



$$l_{g_1 i_1} = 0$$

$$\tau_{g_1 i_1} = 0$$

$$l_{g_2 i_1} = 0$$

$$y_{vr} \Rightarrow \sum_{g \in \mathcal{G}} l_{gr} \leq Q_v \quad \forall v \in \mathcal{V}, \forall r \in \mathcal{R} \quad (34)$$

$$\sum_{g \in \mathcal{G}} l_{go} \leq Q_v \quad \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \quad (35)$$

$$x_{vj} \Rightarrow l_{gj} = 0 \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall (v, j) \in \mathcal{A}, v \in \mathcal{V}, j \neq o_{vg}^1 \quad (36)$$

$$x_{iv} \Rightarrow l_{gi} = 0 \quad \forall g \in \mathcal{G}^{\text{pickup}}, \forall (i, v) \in \mathcal{A}, i \neq o_{vg}^1, v \in \mathcal{V} \quad (37)$$

$$x_{rj} \Rightarrow l_{gj} = l_{gr} - q_r \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall (r, j) \in \mathcal{A}, r \in \mathcal{R}_g, j \notin \mathcal{V} \quad (38)$$

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$$l_{gr} \in \left\{ q_r, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall r \in \mathcal{R}_g \quad (41)$$

$$l_{gr} \in \left\{ 0, \dots, \max_{v \in \mathcal{V}} Q_v - q_r \right\} \quad \forall g \in \mathcal{G}^{\text{pickup}}, \forall r \in \mathcal{R}_g \quad (42)$$

$$l_{gi} \in \left\{ 0, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (43)$$

$$l_{go} \in \{0, \dots, Q_v\} \quad \forall v \in \mathcal{V}, \forall g \in \mathcal{G}, \forall o \in \mathcal{O}_{vg}. \quad (44)$$

$$\tau_{go} = 0 \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall o \in \mathcal{O}_g \quad (45)$$

$$x_{ij} \Rightarrow \tau_{gj} = \tau_{gi} + \text{Start}(\delta_j) - \text{Start}(\delta_i) \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall (i, j) \in \mathcal{A}, j \notin \mathcal{O}_g \quad (46)$$

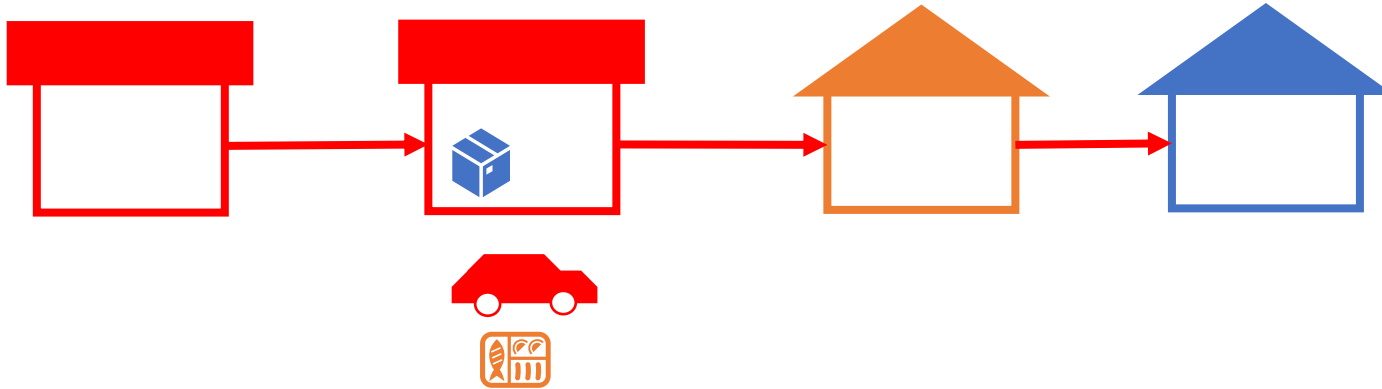
$$\tau_{gr} \in \{0, \dots, T_r\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R}_g \quad (47)$$

$$\tau_{gr} \in \left\{ 0, \dots, b_r - \min_{d \in \mathcal{D}} a_d \right\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (48)$$

$$\tau_{go} \in \{0, \dots, \min\{b_g, b_v\} - a_v\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \setminus \mathcal{O}_{vg}. \quad (49)$$

# Load and Transportation Time Variables

- $l_{gi}$ : load of commodity  $g$  when arriving at location  $i$
- $\tau_{gi}$ : transportation time of commodity  $g$  when arriving at location  $i$



$$\begin{aligned}
 l_{g_1 i_1} &= 0 & l_{g_1 i_2} &= 2 \\
 \tau_{g_1 i_1} &= 0 & \tau_{g_1 i_2} &= 5 \text{ min} \\
 l_{g_2 i_1} &= 0 & l_{g_2 i_2} &= 0
 \end{aligned}$$

$$y_{vr} \Rightarrow \sum_{g \in \mathcal{G}} l_{gr} \leq Q_v \quad \forall v \in \mathcal{V}, \forall r \in \mathcal{R} \quad (34)$$

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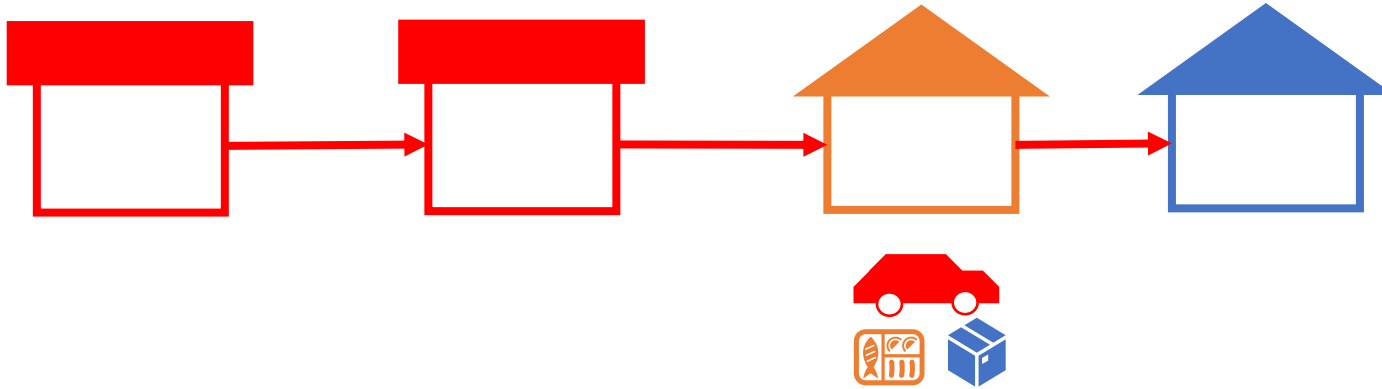
$$\tau_{gr} \in \{0, \dots, T_r\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R}_g \quad (47)$$

$$\tau_{gr} \in \left\{ 0, \dots, b_r - \min_{d \in \mathcal{D}} a_d \right\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (48)$$

$$\tau_{go} \in \{0, \dots, \min\{b_g, b_v\} - a_v\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \setminus \mathcal{O}_{vg}. \quad (49)$$

# Load and Transportation Time Variables

- $l_{gi}$ : load of commodity  $g$  when arriving at location  $i$
- $\tau_{gi}$ : transportation time of commodity  $g$  when arriving at location  $i$



$$\begin{array}{lll}
 l_{g_1 i_1} = 0 & l_{g_1 i_2} = 2 & l_{g_1 i_3} = 2 \\
 \tau_{g_1 i_1} = 0 & \tau_{g_1 i_2} = 5 \text{ min} & \tau_{g_1 i_3} = 15 \text{ min} \\
 l_{g_2 i_1} = 0 & l_{g_2 i_2} = 0 & l_{g_2 i_3} = 5
 \end{array}$$

$$y_{vr} \Rightarrow \sum_{g \in \mathcal{G}} l_{gr} \leq Q_v \quad \forall v \in \mathcal{V}, \forall r \in \mathcal{R} \quad (34)$$

$$\sum_{g \in \mathcal{G}} l_{go} \leq Q_v \quad \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \quad (35)$$

$$x_{vj} \Rightarrow l_{gj} = 0 \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall (v, j) \in \mathcal{A}, v \in \mathcal{V}, j \neq o_{vg}^1 \quad (36)$$

$$x_{iv} \Rightarrow l_{gi} = 0 \quad \forall g \in \mathcal{G}^{\text{pickup}}, \forall (i, v) \in \mathcal{A}, i \neq o_{vg}^1, v \in \mathcal{V} \quad (37)$$

$$x_{rj} \Rightarrow l_{gj} = l_{gr} - q_r \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall (r, j) \in \mathcal{A}, r \in \mathcal{R}, j \notin \mathcal{V} \quad (38)$$

$$x_{rj} \Rightarrow l_{gj} = l_{gr} + q_r \quad \forall g \in \mathcal{G}^{\text{pickup}}, \forall (r, j) \in \mathcal{A}, r \in \mathcal{R}, j \notin \mathcal{V} \quad (39)$$

$$x_{ij} \Rightarrow l_{gj} = l_{gi} \quad \forall g \in \mathcal{G}, \forall (i, j) \in \mathcal{A}, i \notin \mathcal{R}_g \cup \mathcal{O}_g, j \notin \mathcal{V} \quad (40)$$

$$l_{gr} \in \left\{ q_r, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall r \in \mathcal{R}_g \quad (41)$$

$$l_{gr} \in \left\{ 0, \dots, \max_{v \in \mathcal{V}} Q_v - q_r \right\} \quad \forall g \in \mathcal{G}^{\text{pickup}}, \forall r \in \mathcal{R}_g \quad (42)$$

$$l_{gi} \in \left\{ 0, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (43)$$

$$l_{go} \in \{0, \dots, Q_v\} \quad \forall v \in \mathcal{V}, \forall g \in \mathcal{G}, \forall o \in \mathcal{O}_{vg} \quad (44)$$

$$\tau_{go} = 0 \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall o \in \mathcal{O}_g \quad (45)$$

$$x_{ij} \Rightarrow \tau_{gj} = \tau_{gi} + \text{Start}(\delta_j) - \text{Start}(\delta_i) \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall (i, j) \in \mathcal{A}, j \notin \mathcal{O}_g \quad (46)$$

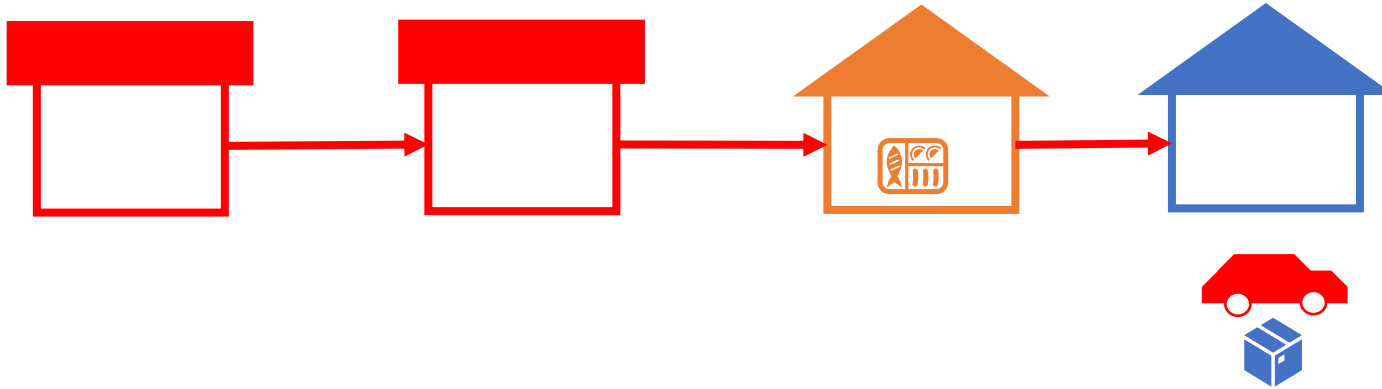
$$\tau_{gr} \in \{0, \dots, T_r\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R}_g \quad (47)$$

$$\tau_{gr} \in \left\{ 0, \dots, b_r - \min_{d \in \mathcal{D}} a_d \right\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (48)$$

$$\tau_{go} \in \{0, \dots, \min\{b_g, b_v\} - a_v\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \setminus \mathcal{O}_{vg} \quad (49)$$

# Load and Transportation Time Variables

- $l_{gi}$ : load of commodity  $g$  when arriving at location  $i$
- $\tau_{gi}$ : transportation time of commodity  $g$  when arriving at location  $i$



|                      |                                  |                                   |                   |
|----------------------|----------------------------------|-----------------------------------|-------------------|
| $l_{g_1 i_1} = 0$    | $l_{g_1 i_2} = 2$                | $l_{g_1 i_3} = 2$                 | $l_{g_1 i_4} = 0$ |
| $\tau_{g_1 i_1} = 0$ | $\tau_{g_1 i_2} = 5 \text{ min}$ | $\tau_{g_1 i_3} = 15 \text{ min}$ |                   |
| $l_{g_2 i_1} = 0$    | $l_{g_2 i_2} = 0$                | $l_{g_2 i_3} = 5$                 | $l_{g_2 i_4} = 5$ |

$$y_{vr} \Rightarrow \sum_{g \in \mathcal{G}} l_{gr} \leq Q_v \quad \forall v \in \mathcal{V}, \forall r \in \mathcal{R} \quad (34)$$

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$$x_{ij} \Rightarrow l_{gj} = l_{gi} \quad \forall g \in \mathcal{G}, \forall (i, j) \in \mathcal{A}, i \notin \mathcal{R}_g \cup \mathcal{O}_g, j \notin \mathcal{V} \quad (40)$$

$$l_{gr} \in \left\{ q_r, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}^{\text{delivery}}, \forall r \in \mathcal{R}_g \quad (41)$$

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$$l_{gi} \in \left\{ 0, \dots, \max_{v \in \mathcal{V}} Q_v \right\} \quad \forall g \in \mathcal{G}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (43)$$

$$l_{go} \in \{0, \dots, Q_v\} \quad \forall v \in \mathcal{V}, \forall g \in \mathcal{G}, \forall o \in \mathcal{O}_{vg}. \quad (44)$$

$$\tau_{go} = 0 \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall o \in \mathcal{O}_g \quad (45)$$

$$x_{ij} \Rightarrow \tau_{gj} = \tau_{gi} + \text{Start}(\delta_j) - \text{Start}(\delta_i) \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall (i, j) \in \mathcal{A}, j \notin \mathcal{O}_g \quad (46)$$

$$\tau_{gr} \in \{0, \dots, T_r\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R}_g \quad (47)$$

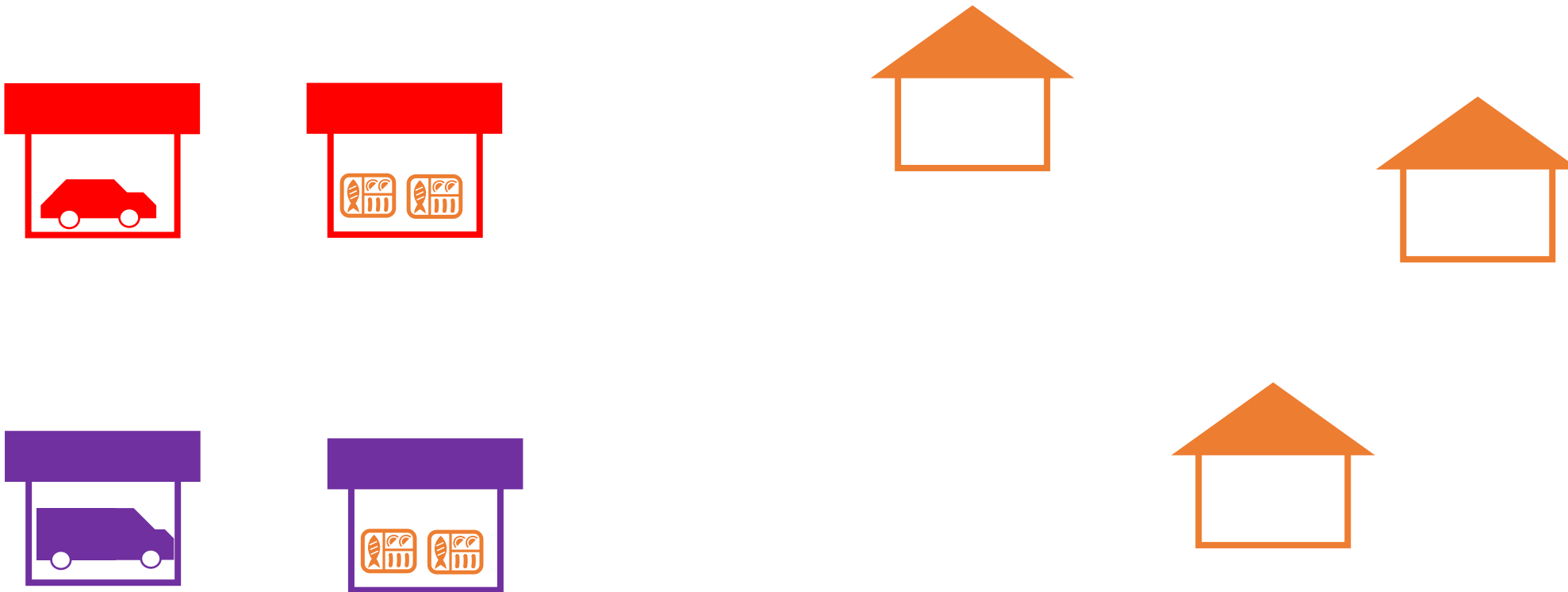
$$\tau_{gr} \in \left\{ 0, \dots, b_r - \min_{d \in \mathcal{D}} a_d \right\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall r \in \mathcal{R} \setminus \mathcal{R}_g \quad (48)$$

$$\tau_{go} \in \{0, \dots, \min\{b_g, b_v\} - a_v\} \quad \forall g \in \mathcal{G}^{\text{perishable}}, \forall v \in \mathcal{V}, \forall o \in \mathcal{O}_v \setminus \mathcal{O}_{vg}. \quad (49)$$

# Iterative Warm-Starting

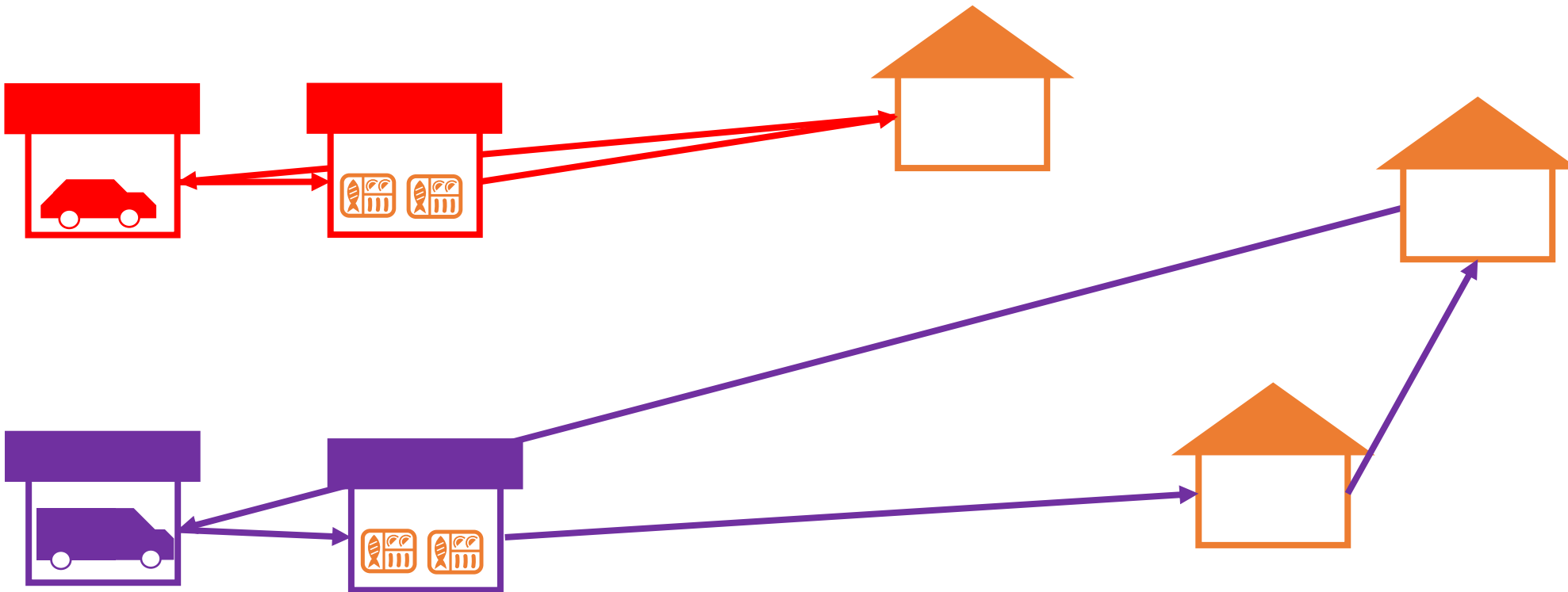
# Iterative Warm-Starting

1. Solve a model considering only one commodity
2. Repeat: add one commodity and solve the resulting model using the solution for the previous model as the initial solution



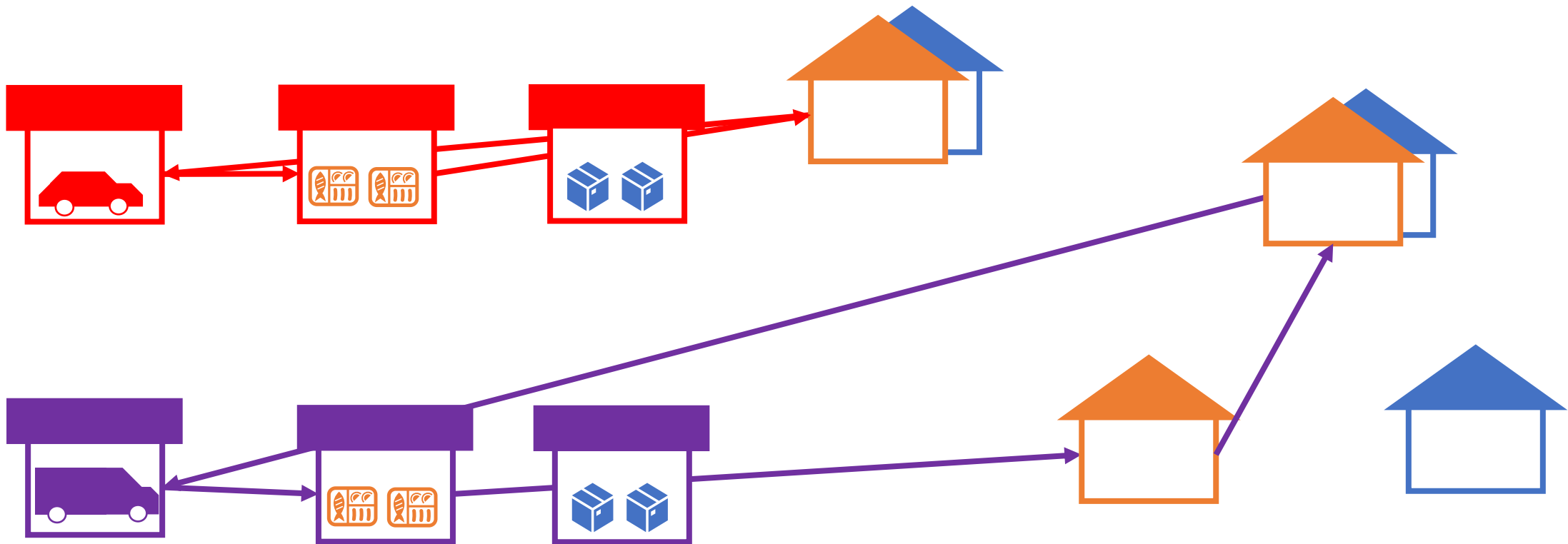
# Iterative Warm-Starting

1. Solve a model considering only one commodity
2. Repeat: add one commodity and solve the resulting model using the solution for the previous model as the initial solution



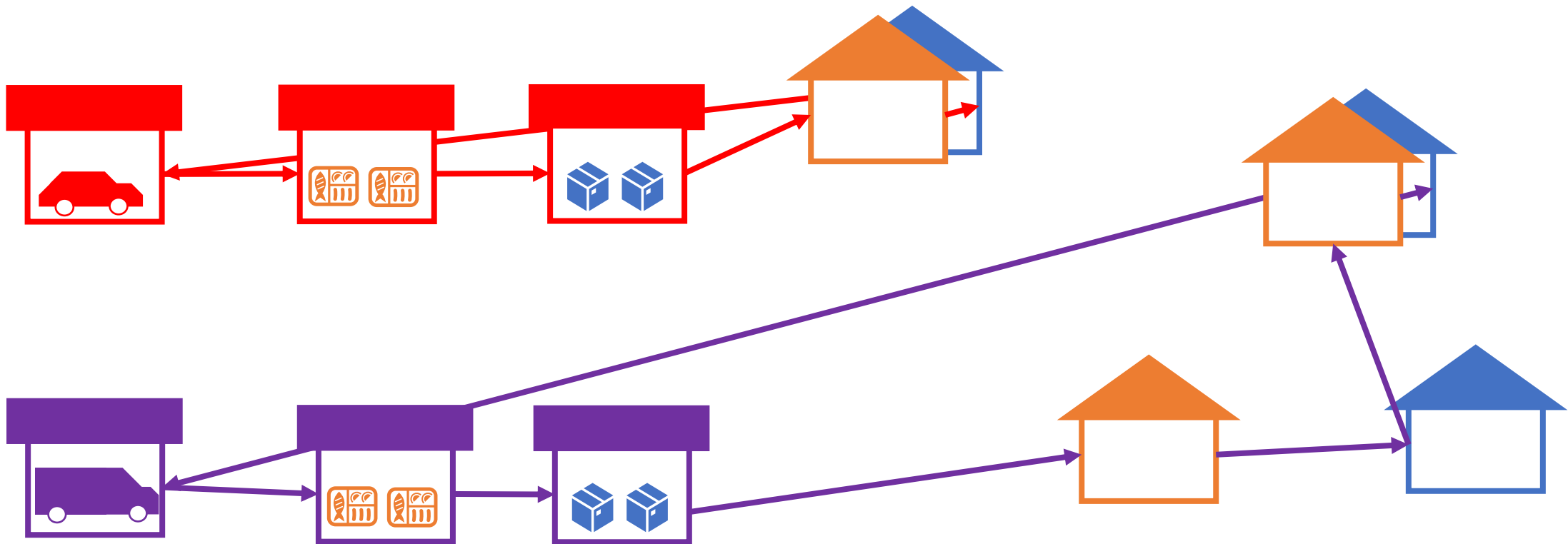
# Iterative Warm-Starting

1. Solve a model considering only one commodity
2. Repeat: add one commodity and solve the resulting model using the solution for the previous model as the initial solution



# Iterative Warm-Starting

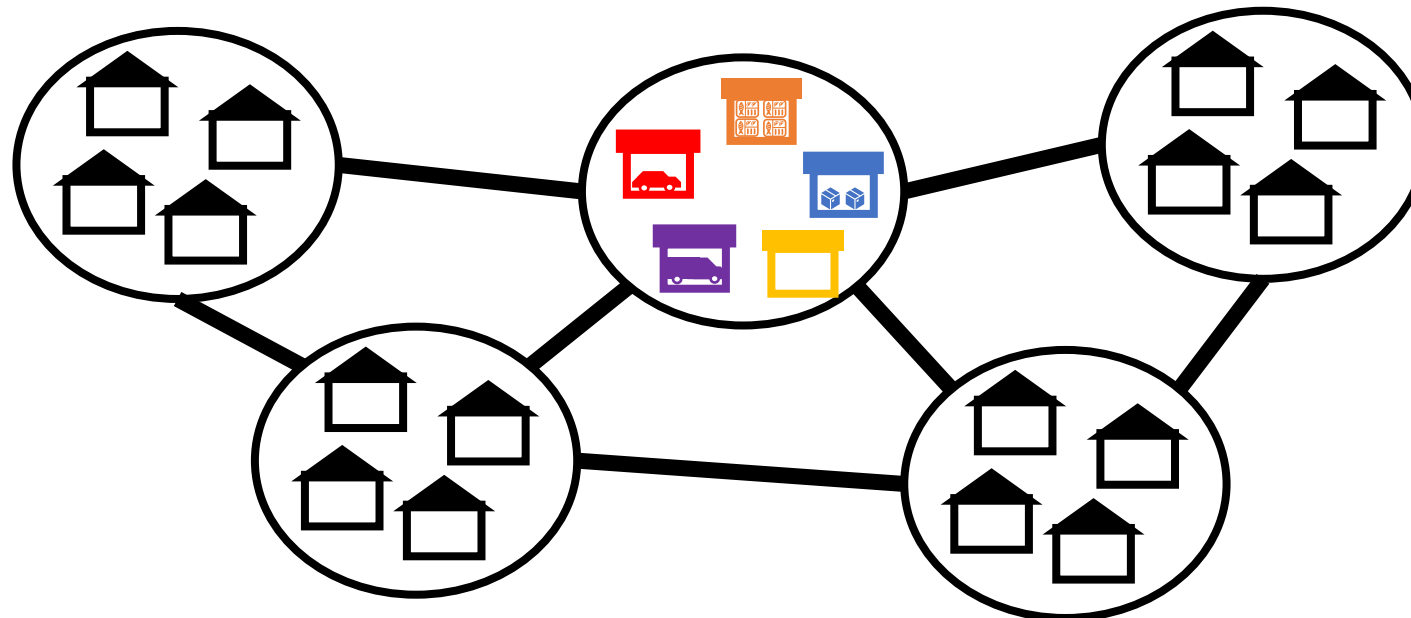
1. Solve a model considering only one commodity
2. Repeat: add one commodity and solve the resulting model using the solution for the previous model as the initial solution



# Experimental Evaluation

# Synthetic Benchmark Instances

- Depots and centers in the central area and customers in rural areas
- Road network in a mesh topology
- 3 commodities: (1) perishable delivery, (2) delivery, (3) pickup, prioritized in this order
- Each rural area has 11 requests in total on average



# Experimental Settings

- Google OR-Tools CP-SAT for CP
- MIP model based on a similar idea as a baseline, solved with Gurobi
- 24 threads and 15-minutes
- Iterative warm-starting uses 5 minute for each model
- 5 instances for each parameter combination

# CP with and without Iterative Warm-Starting

| #areas | #drivers | No Circuit  |             |             | No Circuit Warm |             |             |
|--------|----------|-------------|-------------|-------------|-----------------|-------------|-------------|
|        |          | 1           | 2           | 3           | 1               | 2           | 3           |
| 4      | 3        | 0.89        | 0.84        | 0.46        | <b>0.93</b>     | <b>0.85</b> | 0.43        |
| 8      | 6        | 0.58        | 0.80        | 0.50        | 0.73            | 0.69        | 0.03        |
| 12     | 9        | 0.56        | 0.57        | 0.22        | 0.67            | 0.50        | 0.07        |
| 16     | 12       | 0.41        | 0.38        | 0.12        | <b>0.62</b>     | 0.28        | 0.01        |
| 4      | 6        | <b>1.00</b> | <b>1.00</b> | 0.94        | <b>1.00</b>     | <b>1.00</b> | 0.94        |
| 8      | 12       | 0.91        | 0.94        | 0.60        | <b>1.00</b>     | <b>0.94</b> | 0.41        |
| 12     | 18       | 0.81        | 0.69        | 0.25        | <b>0.98</b>     | 0.60        | 0.17        |
| 16     | 24       | 0.64        | 0.38        | 0.06        | <b>0.88</b>     | 0.29        | 0.03        |
| 4      | 12       | <b>1.00</b> | <b>1.00</b> | <b>1.00</b> | <b>1.00</b>     | <b>1.00</b> | <b>1.00</b> |
| 8      | 24       | <b>1.00</b> | 0.98        | 0.89        | <b>1.00</b>     | 0.98        | 0.88        |
| 12     | 36       | 0.94        | 0.79        | 0.42        | <b>1.00</b>     | 0.71        | 0.21        |
| 16     | 48       | 0.61        | 0.27        | 0.06        | <b>1.00</b>     | 0.26        | 0.03        |

Average ratio of served requests for each commodity

# CP with and without the Circuit Constraint

| #areas | #drivers | Circuit Warm |             |             | No Circuit Warm |             |             |
|--------|----------|--------------|-------------|-------------|-----------------|-------------|-------------|
|        |          | 1            | 2           | 3           | 1               | 2           | 3           |
| 4      | 3        | <b>0.93</b>  | 0.78        | 0.34        | <b>0.93</b>     | <b>0.85</b> | 0.43        |
| 8      | 6        | <b>0.75</b>  | 0.68        | 0.10        | 0.73            | 0.69        | 0.03        |
| 12     | 9        | <b>0.69</b>  | 0.44        | 0.00        | 0.67            | 0.50        | 0.07        |
| 16     | 12       | 0.57         | 0.19        | 0.02        | <b>0.62</b>     | 0.28        | 0.01        |
| 4      | 6        | <b>1.00</b>  | <b>1.00</b> | <b>0.96</b> | <b>1.00</b>     | <b>1.00</b> | 0.94        |
| 8      | 12       | 0.99         | 0.85        | 0.28        | <b>1.00</b>     | <b>0.94</b> | 0.41        |
| 12     | 18       | <b>0.98</b>  | 0.50        | 0.07        | <b>0.98</b>     | 0.60        | 0.17        |
| 16     | 24       | 0.84         | 0.17        | 0.05        | <b>0.88</b>     | 0.29        | 0.03        |
| 4      | 12       | <b>1.00</b>  | <b>1.00</b> | <b>1.00</b> | <b>1.00</b>     | <b>1.00</b> | <b>1.00</b> |
| 8      | 24       | <b>1.00</b>  | <b>0.99</b> | 0.65        | <b>1.00</b>     | 0.98        | 0.88        |
| 12     | 36       | <b>1.00</b>  | 0.67        | 0.05        | <b>1.00</b>     | 0.71        | 0.21        |
| 16     | 48       | 0.97         | 0.14        | 0.02        | <b>1.00</b>     | 0.26        | 0.03        |

Average ratio of served requests for each commodity

# CP vs. MIP

| #areas | #drivers | MIP Warm    |             |      | No Circuit Warm |             |             |
|--------|----------|-------------|-------------|------|-----------------|-------------|-------------|
|        |          | 1           | 2           | 3    | 1               | 2           | 3           |
| 4      | 3        | 0.89        | 0.70        | 0.14 | <b>0.93</b>     | <b>0.85</b> | 0.43        |
| 8      | 6        | 0.63        | 0.44        | 0.04 | <b>0.73</b>     | 0.69        | 0.03        |
| 12     | 9        | 0.58        | 0.06        | 0.00 | <b>0.67</b>     | 0.50        | 0.07        |
| 16     | 12       | 0.41        | 0.02        | 0.00 | <b>0.62</b>     | 0.28        | 0.01        |
| 4      | 6        | <b>1.00</b> | 0.99        | 0.75 | <b>1.00</b>     | <b>1.00</b> | <b>0.94</b> |
| 8      | 12       | 0.94        | 0.31        | 0.09 | <b>1.00</b>     | <b>0.94</b> | 0.41        |
| 12     | 18       | 0.88        | 0.07        | 0.00 | <b>0.98</b>     | 0.60        | 0.17        |
| 16     | 24       | 0.60        | 0.00        | 0.00 | <b>0.88</b>     | 0.29        | 0.03        |
| 4      | 12       | <b>1.00</b> | <b>1.00</b> | 0.90 | <b>1.00</b>     | <b>1.00</b> | <b>1.00</b> |
| 8      | 24       | <b>1.00</b> | 0.63        | 0.15 | <b>1.00</b>     | <b>0.98</b> | 0.88        |
| 12     | 36       | <b>1.00</b> | 0.02        | 0.00 | <b>1.00</b>     | 0.71        | 0.21        |
| 16     | 48       | 0.77        | 0.00        | 0.00 | <b>1.00</b>     | 0.26        | 0.03        |

Average ratio of served requests for each commodity

# Conclusion and Future Work

CP is useful for solving a complicated routing problem

Future work

- Real-world deployment
- Large neighborhood search

# Vehicle and Driver Constraints

$$\text{AtMostOne}(\{y_{rv} \mid v \in \mathcal{V}\}) \quad \forall r \in \mathcal{R} \quad (19)$$

$$\text{Pres}(\delta_r) \implies \text{AtLeastOne}(\{y_{rv} \mid v \in \mathcal{V}\}) \quad \forall r \in \mathcal{R} \quad (20)$$

$$x_{vr} \implies y_{rv} \quad \forall (v, r) \in \mathcal{A}, v \in \mathcal{V}, r \in \mathcal{R}^{\text{pickup}} \quad (21)$$

$$x_{rv} \implies y_{rv} \quad \forall (r, v) \in \mathcal{A}, r \in \mathcal{R}^{\text{delivery}}, v \in \mathcal{V} \quad (22)$$

$$x_{ij} \implies y_{iv} = y_{jv} \quad \forall v \in \mathcal{V}, \forall (i, j) \in \mathcal{A}, i, j \in \mathcal{R} \quad (23)$$

$$x_{ro} \implies y_{rv} \quad \forall v \in \mathcal{V}, \forall (r, o) \in \mathcal{A}, r \in \mathcal{R}, o \in \mathcal{O}_v \quad (24)$$

$$x_{or} \implies y_{rv} \quad \forall v \in \mathcal{V}, \forall (o, r) \in \mathcal{A}, o \in \mathcal{O}_v, r \in \mathcal{R} \quad (25)$$

$$\text{AtMostOne}(\{z_{dv} \mid v \in \mathcal{V}_d\}) \quad \forall d \in \mathcal{D} \quad (27)$$

$$\text{AtMostOne}(\{z_{dv} \mid d \in \mathcal{D}_v\}) \quad \forall v \in \mathcal{V} \quad (28)$$

$$\text{Pres}(\lambda_v) \implies \text{AtLeastOne}(\{z_{vd} \mid d \in \mathcal{D}_v\}) \quad \forall v \in \mathcal{V} \quad (29)$$

$$z_{dv} \implies \text{Start}(\lambda_v) \geq a_d \quad \forall d \in \mathcal{D}, \forall v \in \mathcal{V}_d \quad (30)$$

$$z_{dv} \implies \text{End}(\lambda_v) \leq b_d \quad \forall d \in \mathcal{D}, \forall v \in \mathcal{V}_d \quad (31)$$

$$z_{dv} \implies \text{Length}(\lambda_v) \leq T_d \quad \forall d \in \mathcal{D}, \forall v \in \mathcal{V}_d \quad (32)$$